

Sec 12-3
(Analyzing Data)

1) Measures of Central Tendency

a) Mean (\bar{x})

- Arithmetic average $\bar{x} = \frac{\sum x_i}{n}$
- Affected by extreme values (outliers)

Ex 2, 4, 6 ($\bar{x} = 4$)

2, 4, 20 ($\bar{x} = 8.67$)

b) Median (M)

- Middle value

Ex 2, 4, 6 (M=4)

2, 4, 6, 8 (M=5)

- Not affected by outliers

Ex 2, 4, 20 (M=4)

c) Mode

- Most frequent value (if there is one)

2) Spread (Sec 12-4)



vs

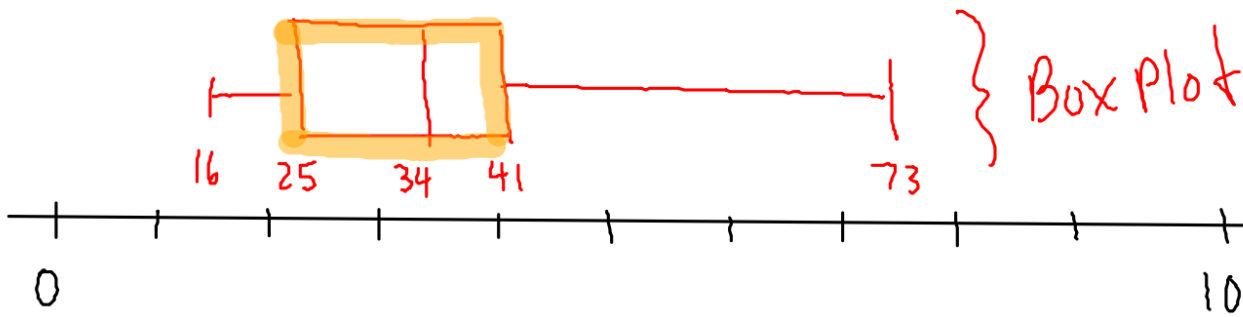


3) 5-Number Summary

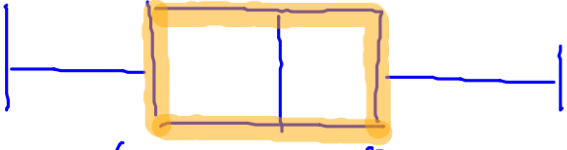
Ex Test Scores ($\bar{x} = 35$):

16 19 24 25 25 25 33 34 34 37 37 40 42 46 49 73

↑ min ↑ $Q_1 = 25$ ↑ $M = 34$ ↑ $Q_3 = 41$ ↑ Max



$$\text{Interquartile Range (IQR)} = 41 - 25 = 16$$

4) Outliers x 

$$Q_1 - 1.5(IQR) > \text{Outlier} > Q_3 + 1.5(IQR)$$

$$25 - 1.5(16) > \text{Outlier} > 41 + 1.5(16)$$

$$1 > \text{Outlier} > 65$$

↗
No Outliers

↖
73 is outlier

Percentiles

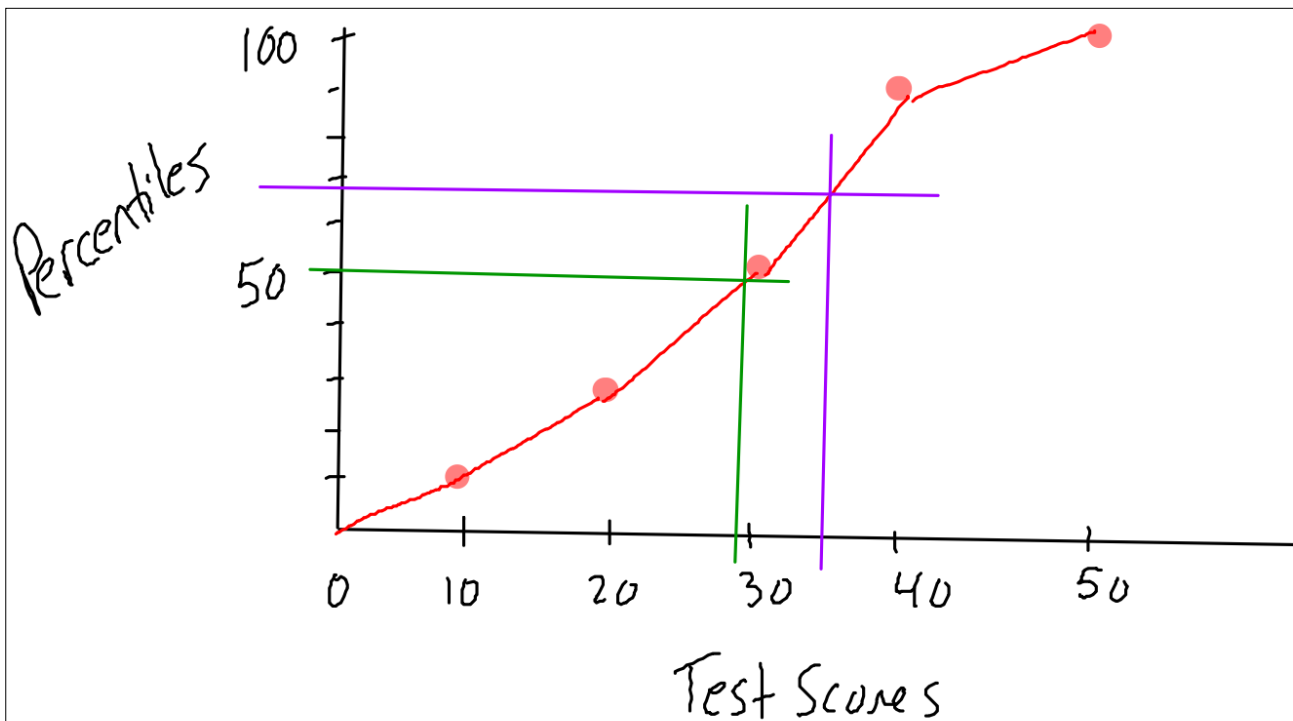
Percent of data that falls below
a certain value

Ex PSAT Score \rightarrow 90th Percentile

90% of students got
less than your score

Ogive (O-Jive)

Test Score	Frequency	Relative Frequency (%)	Relative Cumulative Frequency
0 - 9	4	10%	10%
10 - 19	8	20	30
20 - 29	10	25	55
30 - 39	15	37.5	92.5
40 - 50	3	7.5	100
	<u>40</u>	<u>100%</u>	



28 falls in the 50th Percentile

35 is in the 68th Percentile

Sec 12-4
(Measuring Spread)

Measures of Dispersion

1) Range (max - min)

2) IQR ($Q_3 - Q_1$) \rangle Middle 50% of data

3) Standard Deviation (s)

$$a) s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$b) s = \sqrt{\frac{n \sum x^2 - (\sum x)^2}{n(n-1)}}$$

Ex Find standard deviation of $\{2, 4, 6\}$

a)

$$s = \sqrt{\frac{(2-4)^2 + (4-4)^2 + (6-4)^2}{3-1}}$$

$$\bar{x} = 4$$

$$= \sqrt{\frac{4 + 0 + 4}{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

Ex Find standard deviation of $\{2, 4, 6\}$

$$b) S = \sqrt{\frac{3(2^2 + 4^2 + 6^2) - (2 + 4 + 6)^2}{3(3-1)}}$$

$$= \sqrt{\frac{168 - 144}{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$$

Notes

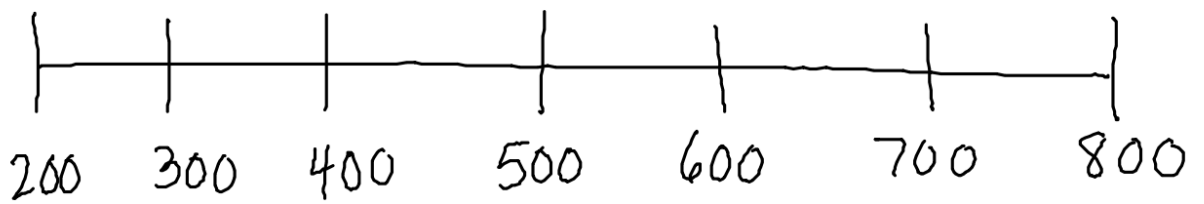
- The bigger s is, the more spread-out the data is from the mean
- s is not resistant (affected by outliers)

Z-Score

The number of standard deviations that a value is from a mean

$$-3 \leq Z \leq 3$$

SAT Scores ($\bar{X} = 500, S = 100$)



Z-score -3 -2 -1 0 1 2 3

$$Z = \frac{X - \bar{X}}{S}$$

Ex Find z-score of 625

$$Z = \frac{625 - 500}{100} = 1.25$$

Ex Find z-score of 310:

$$Z = \frac{310 - 500}{100} = -1.9$$

Sec 12.5

(Working With Samples)

Population

The entire set of people or things (LN students)

Parameter

A number (mean μ , proportion p)
that describes a population

Sample

Subset of population taken randomly

Statistic

Number (mean \bar{x} or proportion \hat{p})

used to describe a sample

Proportion Property

When a large sample of size n is taken from a population then the likely interval for the true population

proportion is: $\hat{p} \pm \frac{1}{\sqrt{n}}$

↑ Margin of Error (MOE)

In a poll of 2750 airline travelers, 138 said they never check their luggage when they fly. Find the sample proportion, the margin of error (MOE) and the interval likely to contain the true population proportion

a) Sample Proportion (\hat{p}) = $\frac{138}{2750} \approx 5\%$

b) MOE = $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{2750}} \approx 2\%$

c) Interval = $(5\% \pm 2\%) = 3\% \text{ and } 7\%$

A poll reports 56% of voters prefer Candidate A with a MOE of 3%. Estimate the number of voters used in the poll.

$$\text{MOE} = \frac{1}{\sqrt{n}}$$

$$\sqrt{n} = \frac{1}{.03}$$

$$\frac{.03}{1} = \frac{1}{\sqrt{n}}$$

$$n = \left(\frac{1}{.03}\right)^2$$

$$.03\sqrt{n} = 1$$

$$n = 1111.1 = 1112 \text{ people}$$

Sec 12-6
(Binomial Setting)

Binomial Setting

- 1) 2 Outcomes (Success / Failure)
- 2) $P(\text{success})$ is constant

Ex MC test with 50 questions

1) 2 outcomes (correct/incorrect)

$$2) P(\text{success}) = \frac{1}{5} = .20$$

Find probability you get exactly 10 questions correct by random guessing.

$$P(X=k) = n C_k p^k (1-p)^{n-k}$$

$\uparrow \quad \frac{n!}{k!(n-k)!}$

$$P(X=10) = {}_{50}C_{10} (.2)^{10} (.8)^{40} \approx .1398$$

A calculator contains 4 batteries. With normal use, each battery has a 90% chance of lasting for one year. What is the probability that all 4 last for one year?

$$P(X=4) = {}_4C_4 (.9)^4 (.1)^0 \approx .6561$$

Find the probability that you get at least 3 questions correct by guessing on a 5-question MC test

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$P(X=3) = {}_5C_3 (.2)^3 (.8)^2 \approx .0512$$

$$P(X=4) = {}_5C_4 (.2)^4 (.8)^1 \approx .0064$$

$$P(X=5) = {}_5C_5 (.2)^5 (.8)^0 \approx \underline{.00032}$$

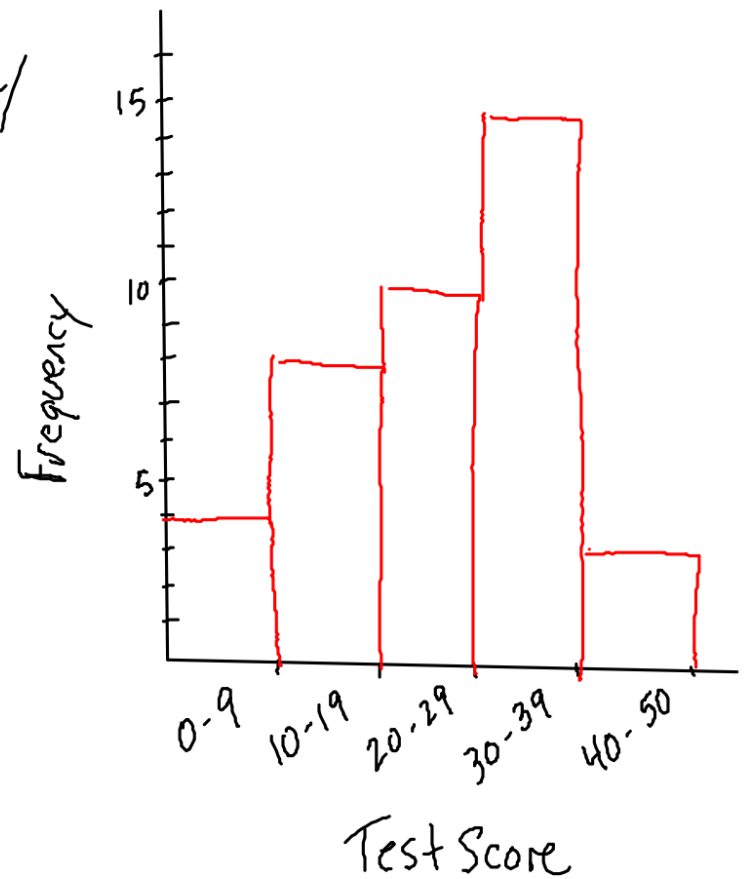
.05792

Sec 12 - 7

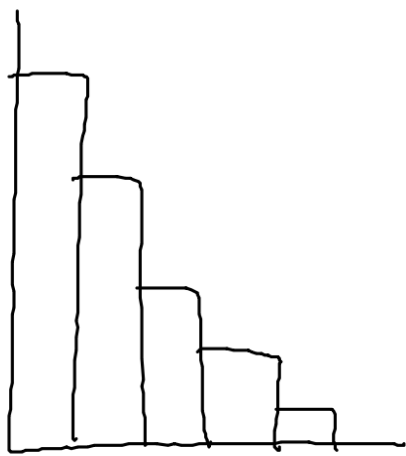
(Normal Distributions)

Histogram - Uses a range of values

<u>Test Score</u>	<u>Frequency</u>
0 - 9	4
10 - 19	8
20 - 29	10
30 - 39	15
40 - 50	3



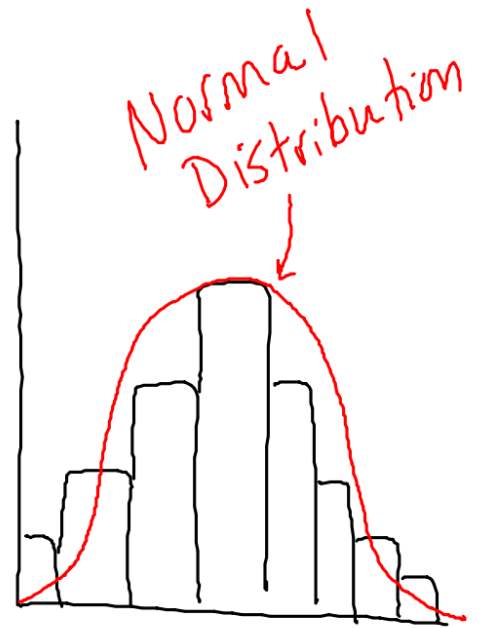
Shapes



Skewed Right



Skewed Left



Symmetric

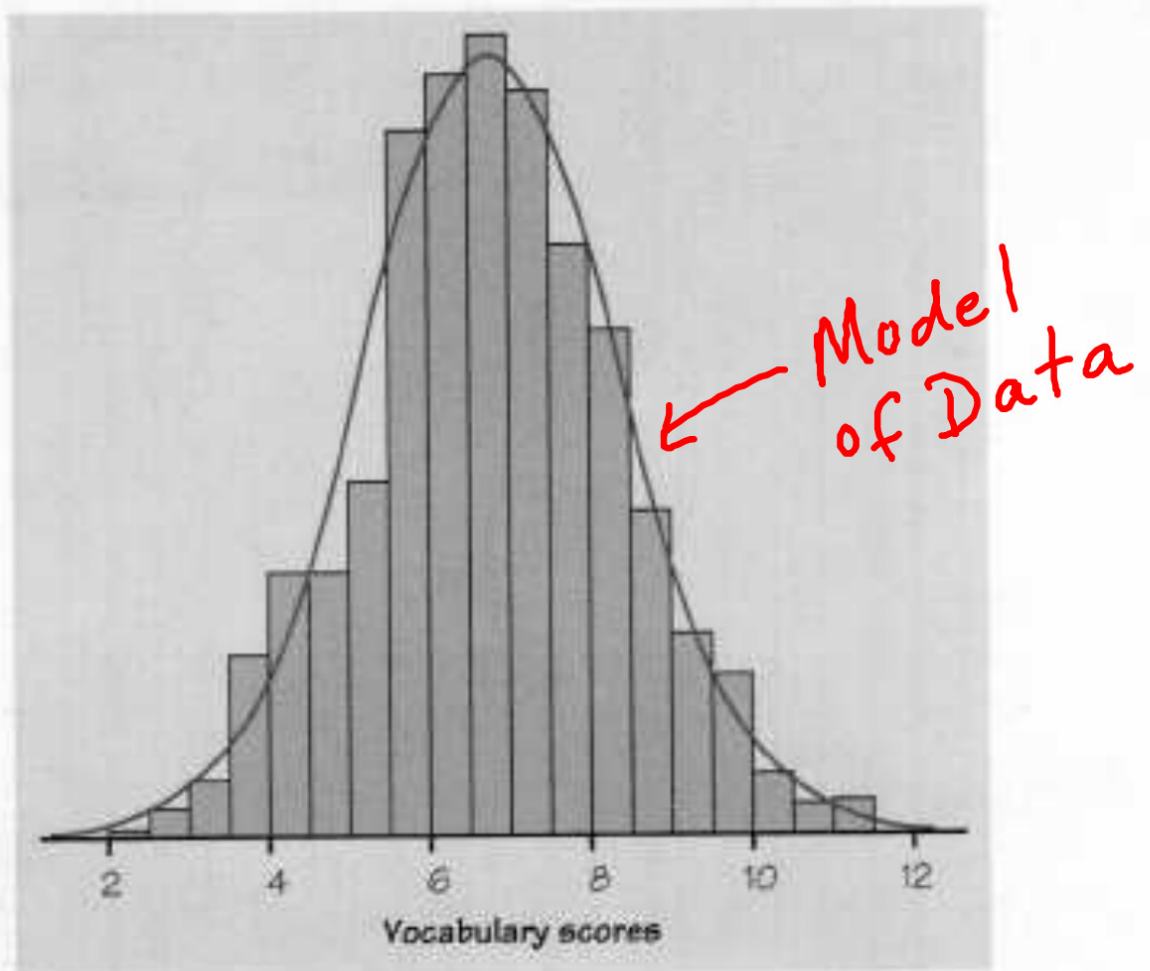


FIGURE 2.2 Histogram of the vocabulary scores of all seventh-grade students in Gary, Indiana. The smooth curve shows the overall shape of the distribution.

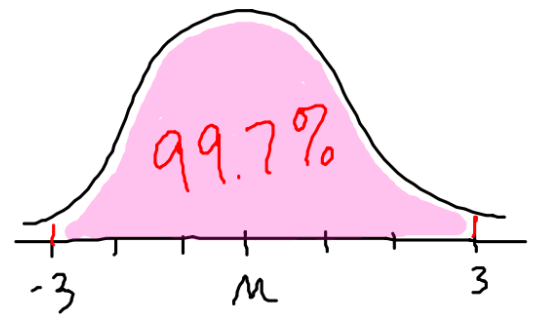
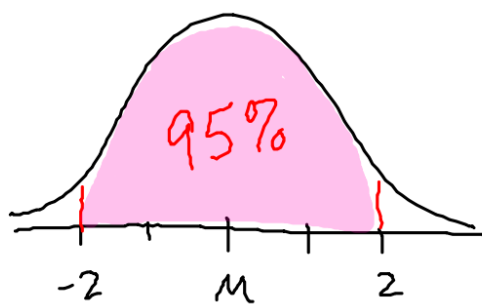
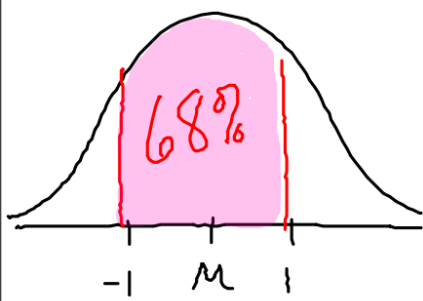
Notation

$$N(\mu, \sigma)$$

↑
mean

↑
standard
deviation

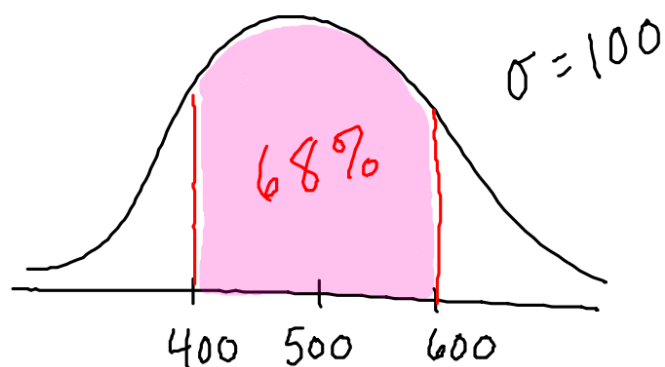
68 - 95 - 99.7 Rule



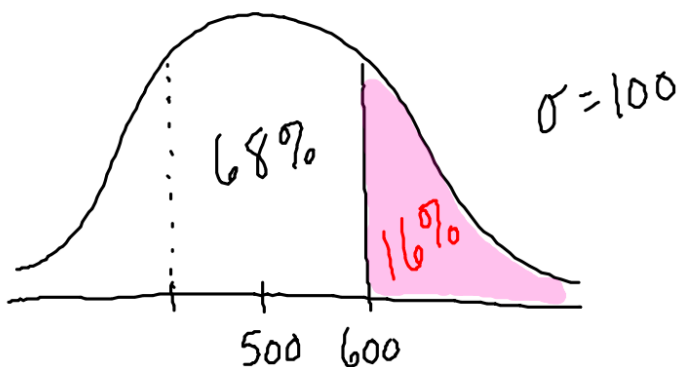
SAT Math Scores $\rightarrow N(500, 100)$

Let X = random SAT Math Score

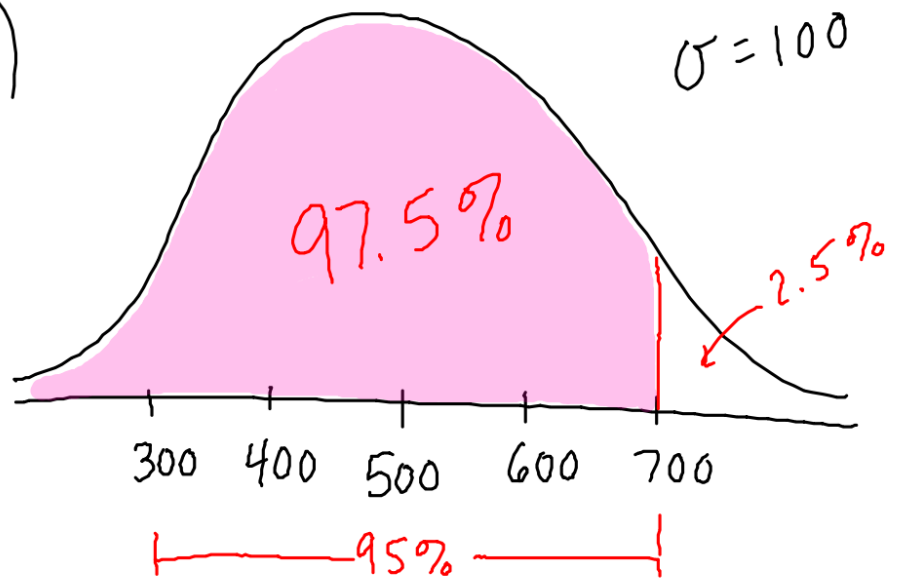
1) $P(400 < X < 600)$



2) $P(X > 600)$

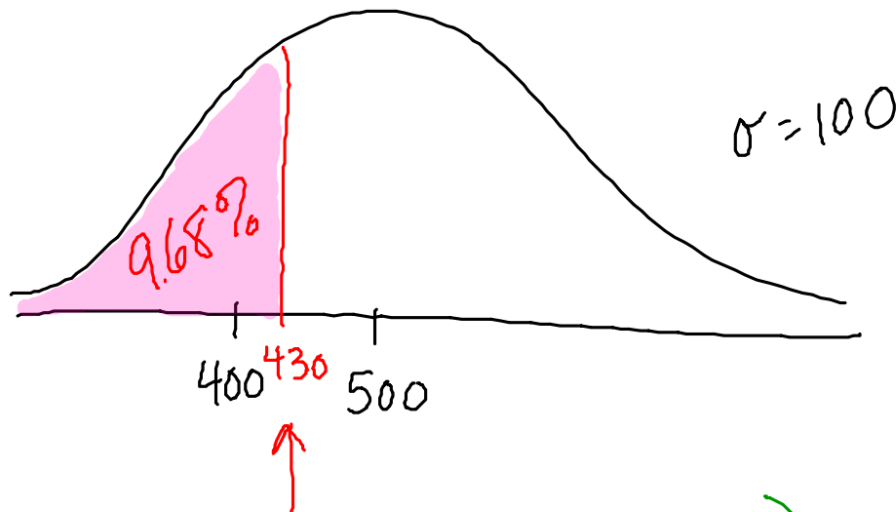


3) $P(X < 700)$



$$4) P(X < 430)$$

} Need to use
Z-scores

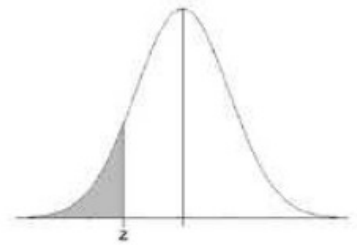


$$Z = \frac{X - \bar{X}}{S} = \frac{430 - 500}{100} = -1.30$$

} See Standard
Normal Cumulative
Probability Table

Standard Normal Cumulative Probability Table

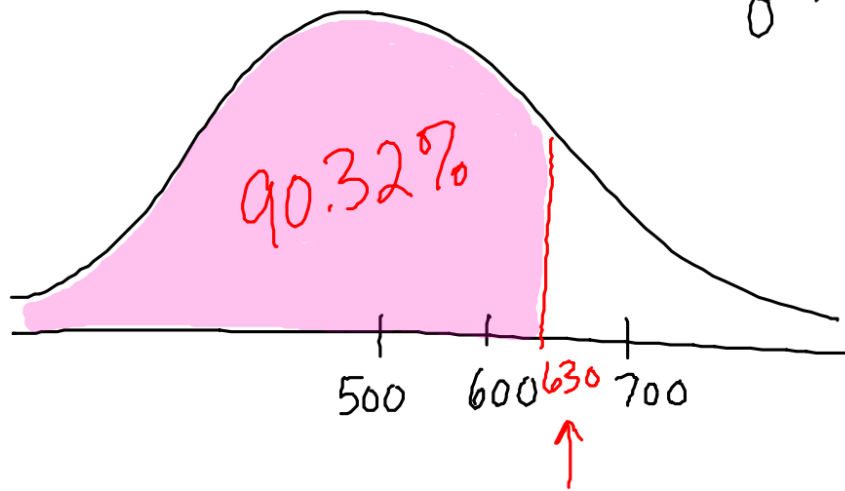
Cumulative probabilities for NEGATIVE z-values are shown in the following table:



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

$$5) P(X < 630)$$

$$\sigma = 100$$



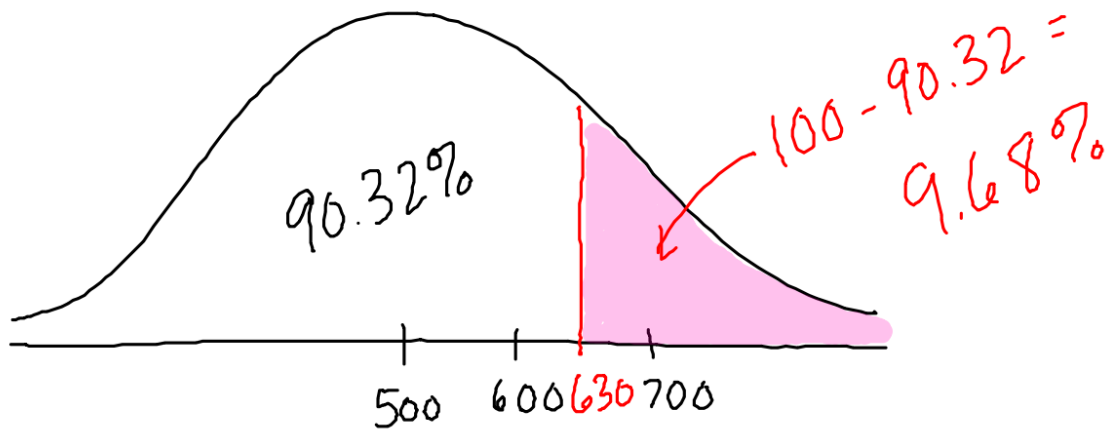
$$Z = \frac{630 - 500}{100} = 1.30$$

A normal distribution curve is shown with a vertical line at a point labeled z on the horizontal axis. The area under the curve to the left of z is shaded gray, representing the cumulative probability $P(Z \leq z)$.

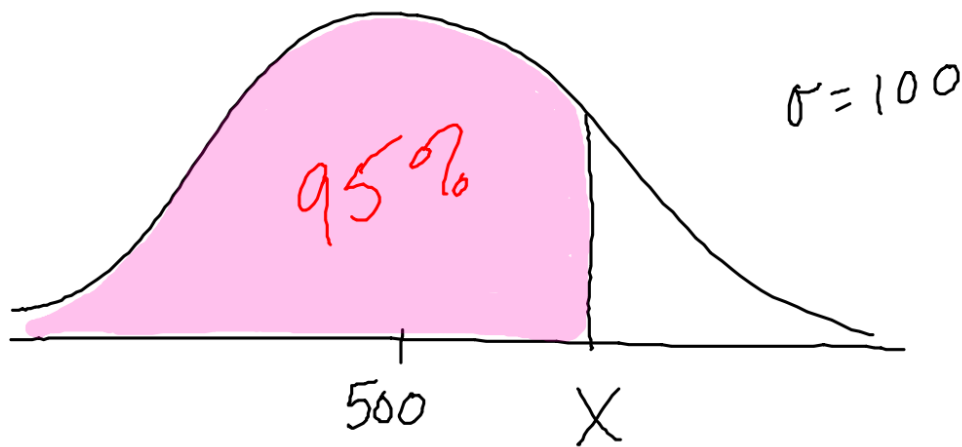
2

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

6) $P(X > 630)$



7) What SAT Math Score puts
Someone in the 95th Percentile ?



a) Find Z-score (Table) corresponding to .9500

$$Z = 1.645$$

b) Use Z-formula to find score:

$$Z = \frac{X - \bar{X}}{s}$$

$$1.645 = X - 500$$

$$664.5 = X = 660$$

↑

SAT Score

$$1.645 = \frac{X - 500}{100}$$

A normal distribution curve is shown with a vertical line at a point labeled z on the horizontal axis. The area under the curve to the left of z is shaded gray, representing the cumulative probability $P(Z \leq z)$.

2

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998