

Sec 7-1

Index  $\sqrt[n]{a}$  Radical

Radicand

Simplify  $\{\text{Reals}\}$

$$\sqrt{64} = 8 \longrightarrow 8^2 = 64$$

$$\sqrt[3]{64} = 4 \longrightarrow 4^3 = 64$$

$$\sqrt[6]{64} = 2 \longrightarrow 2^6 = 64$$

Simplify

$$\sqrt{x^2} = x^1$$

$$\sqrt{x^6} = x^3$$

$$\sqrt{x^8} = x^4$$

$$\sqrt[4]{x^4 y^8} = x^1 y^2$$

Simplify

$$\sqrt[3]{-125x^6} = -5x^2$$

$$\sqrt[4]{-625} = \text{None } \{\text{Reals}\}$$

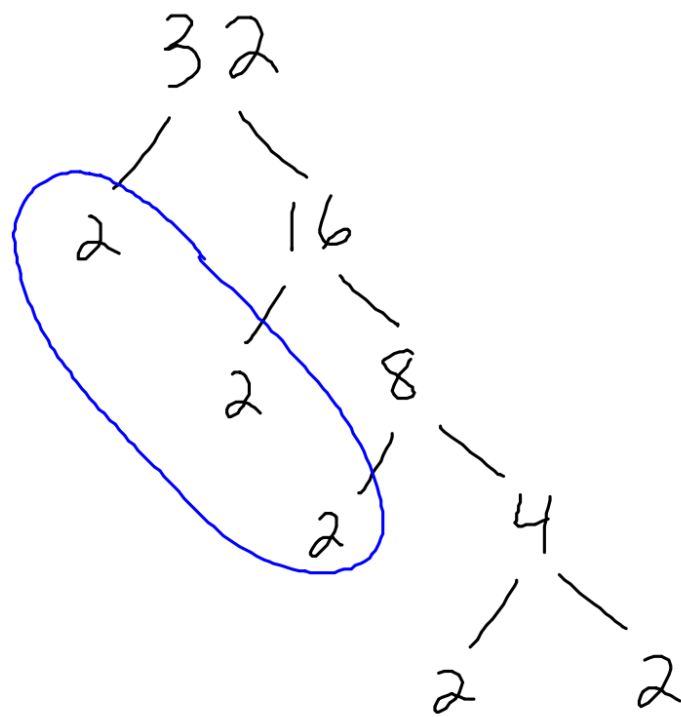
Secs 7-2 and 7-3

# Simplifying Radicals

i) No perfect  $n^{\text{th}}$  factors in radicand

$$\sqrt{24} = \sqrt{4} \sqrt{6} = 2\sqrt{6}$$

$$\sqrt[3]{32x^4} = \sqrt[3]{8x^3} \sqrt[3]{4x} = 2x \sqrt[3]{4x}$$





2) No radicals in denominator

$$\frac{5\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{9}} = \frac{5\sqrt{3}}{3}$$

$$\frac{4\sqrt[3]{36}}{\sqrt[3]{6}\sqrt[3]{36}} = \frac{4\sqrt[3]{36}}{\sqrt[3]{216}} = \frac{4\sqrt[3]{36}}{6} = \frac{2\sqrt[3]{36}}{3}$$

$$\frac{12\sqrt[3]{6}}{\sqrt[3]{36}\sqrt[3]{6}} = \frac{12\sqrt[3]{6}}{\sqrt[3]{216}} = \frac{12\sqrt[3]{6}}{6} = 2\sqrt[3]{6}$$

$$\frac{8}{\sqrt[3]{9x}} \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{8\sqrt[3]{3x^2}}{\sqrt[3]{27x^3}} = \frac{8\sqrt[3]{3x^2}}{3x}$$

$$\frac{(6 + \sqrt{5})(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})} = \frac{6 + 6\sqrt{3} + \sqrt{5} + \sqrt{15}}{1 - \sqrt{9}}$$

Conjugates

$$= \frac{6 + 6\sqrt{3} + \sqrt{5} + \sqrt{15}}{-2}$$

3) No fractions/decimals as radicands

$$\sqrt[3]{\frac{3}{5}} = \frac{\sqrt[3]{3}}{\sqrt[3]{5}} \frac{\sqrt[3]{25}}{\sqrt[3]{25}} = \frac{\sqrt[3]{75}}{\sqrt[3]{125}} = \frac{\sqrt[3]{75}}{5}$$

$$\sqrt{.05} = \sqrt{\frac{5}{100}} = \frac{\sqrt{5}}{\sqrt{100}} = \frac{\sqrt{5}}{10}$$

## Notes

$$(\sqrt{3})(\sqrt{2}) = \sqrt{6}$$

$$(\sqrt[3]{3})(\sqrt{2}) = \sqrt[3]{3} \sqrt{2}$$

$$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3} \quad \left. \vphantom{\begin{matrix} 2\sqrt{3} + 3\sqrt{3} \\ 2\sqrt{3} + 3\sqrt{3} \end{matrix}} \right\} 2x + 3x = 5x$$

# Properties of Exponents

$a^n$

base  $\nearrow$

$\nwarrow$  exponent/power

$$a^0 = 1$$

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$5^0 = 1$$

$$x^2 \cdot x^3 = x^5$$

$$(x^2)^3 = x^6$$

$$\frac{x^5}{x^2} = x^3$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(2x)^3 = 8x^3$$

$$\left(\frac{2}{x}\right)^4 = \frac{16}{x^4}$$

$$5^{-2} = \frac{1}{25}$$

$$\frac{1}{5^{-2}} = 25$$



$$\underline{\text{Ex}} \quad (7a^2)(-2a^{-5})$$

$$= -14a^{-3}$$

$$= \frac{-14}{a^3}$$

$$\underline{\text{Ex}} \quad (-2x^{-1}y^2)^3$$

$$= -8x^{-3}y^6$$

$$= \frac{-8y^6}{x^3}$$

$$\underline{\text{Ex}} \quad \frac{2a^1b^5c^2}{a^3b^1c^2}$$

$$= 2a^{-2}b^4\cancel{c^0}$$

$$= \frac{2b^4}{a^2}$$

Sec 7-4

# Rational Exponents

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Radical Form

$$\sqrt{2^1}$$

$$\sqrt[3]{2^1}$$

$$\sqrt[3]{25} = \sqrt[3]{5^2}$$

$$\frac{1}{\sqrt{3}}$$

Exponential Form

$$2^{\frac{1}{2}}$$

$$2^{\frac{1}{3}}$$

$$5^{\frac{2}{3}}$$

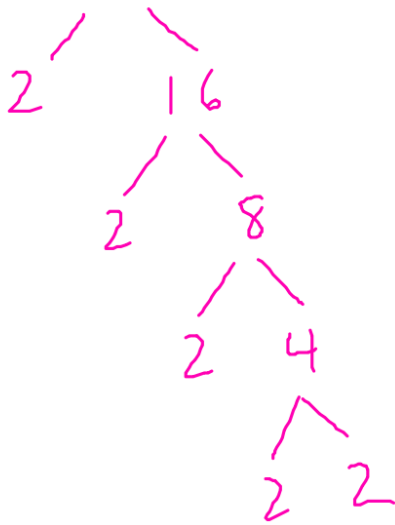
$$\frac{1}{3^{\frac{1}{2}}} = 3^{-\frac{1}{2}}$$

Simplify

$$\begin{aligned}\sqrt{5} \cdot \sqrt[3]{5} &= 5^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \\&= 5^{\frac{3}{2} + \frac{1}{3}} \\&= 5^{\frac{5}{6}} \\&= \sqrt[6]{5^5} \\&= \sqrt[6]{3125}\end{aligned}$$

Simplify

$$32^{\frac{3}{5}} = (2^5)^{\frac{3}{5}} = 2^3 = 8$$





Simplify

$$25^{-\frac{3}{2}} = \frac{1}{125}$$

$$25^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = 5^3 = 125$$

Simplify

$$(8x^{15})^{-\frac{1}{3}} = \frac{1}{2x^5}$$

→  $(8x^{15})^{\frac{1}{3}} = (2^3 x^{15})^{\frac{1}{3}} = 2x^5$

4) Index should be as small as possible

$$\sqrt[4]{9} = \sqrt[4]{3^2} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}$$

$$\sqrt[6]{16} = \sqrt[6]{2^4} = 2^{\frac{4}{6}} = 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}$$

Sec 7-5

Solve  $\sqrt{5x+1} - 6 = 0$

$$(\sqrt{5x+1})^2 = (6)^2 \quad \text{Isolate } \sqrt{\phantom{x}}$$

$$5x + 1 = 36 \quad \text{"Square" Each Side}$$

$$5x = 35$$

$$x = 7$$

} Solve & Check

$$\sqrt{5(7)+1} - 6 = \sqrt{36} - 6 = 6 - 6 = 0 \checkmark$$

$$2(x+3)^{\frac{3}{2}} = 54$$

$$(x+3)^{\frac{3}{2} \cdot \frac{2}{3}} = (27)^{\frac{2}{3}}$$

$$x+3 = 9$$

$$x = 6$$

$$2(6+3)^{3/2} = 2(9)^{3/2} = 2(27) = 54 \checkmark$$

$$\sqrt{5x-1} + 3 = x$$

$$(\sqrt{5x-1})^2 = (x-3)^2 \quad \text{FOIL!}$$

$$5x-1 = x^2-6x+9$$

$$0 = x^2-11x+10$$

$$0 = (x-10)(x-1)$$

$$x = 10 \text{ or } \times \text{ Extraneous}$$

$$\sqrt{5(10)-1} + 3 = \sqrt{49} + 3 = 7 + 3 = 10 \checkmark$$

$$\sqrt{5(1)-1} + 3 = \sqrt{4} + 3 = 2 + 3 = 5 \times$$

$$\sqrt{x+10} + \sqrt{3-x} = 5$$

$$(\sqrt{x+10})^2 = (5 - \sqrt{3-x})^2 (5 - \sqrt{3-x})$$

$$x+10 = 25 - 5\sqrt{3-x} - 5\sqrt{3-x} + (3-x)$$

$$x+10 = 28 - x - 10\sqrt{3-x}$$

$$(2x-18)^2 = (-10\sqrt{3-x})^2$$

$$4x^2 - 72x + 324 = 100(3-x)$$

$$4x^2 - 72x + 324 = 300 - 100x$$

$$4x^2 + 28x + 24 = 0$$

$$x^2 + 7x + 6 = 0$$

$$(x+6)(x+1) = 0$$

$$\boxed{x = -6, -1}$$



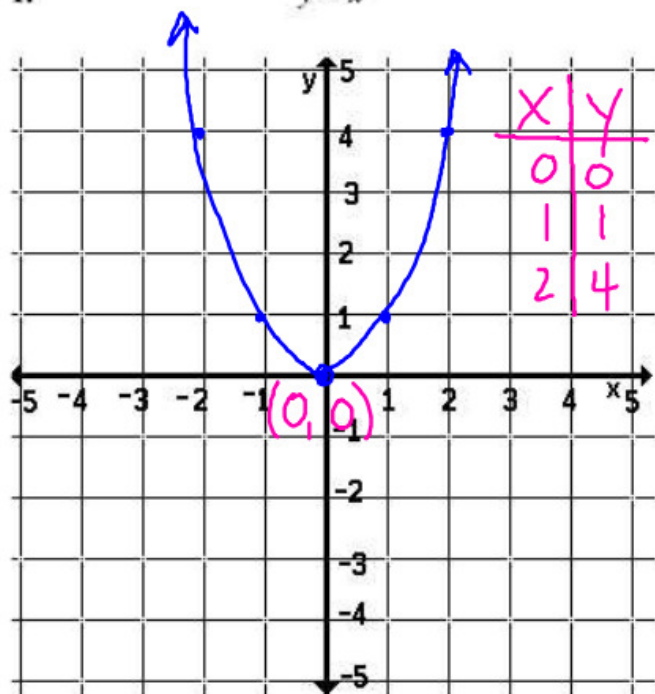
Sec 7-8

# GRAPHING RADICAL EQUATIONS

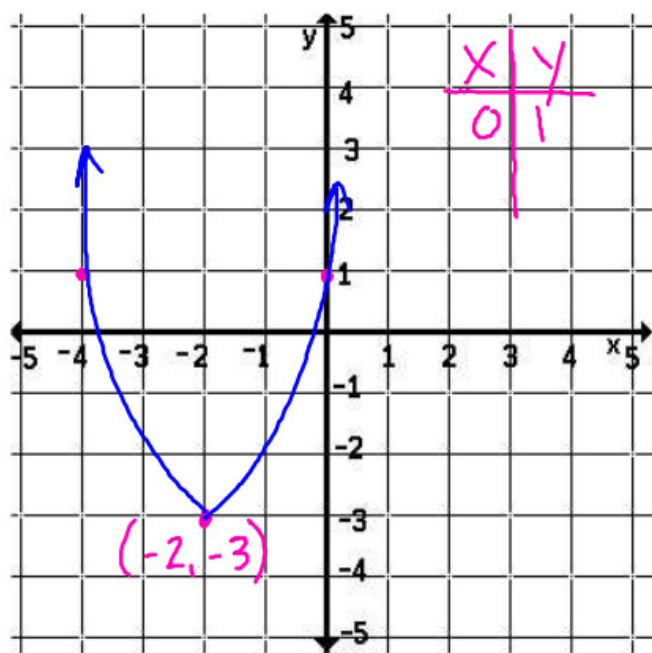
$$y = a(x-h)^2 + k$$

1.

$$y = x^2$$

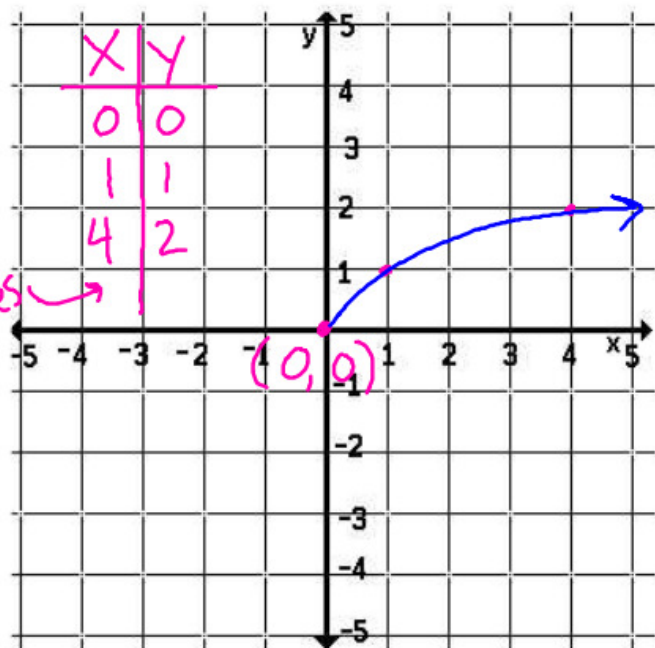


$$y = (x+2)^2 - 3 \quad \text{Vertex } (-2, -3)$$

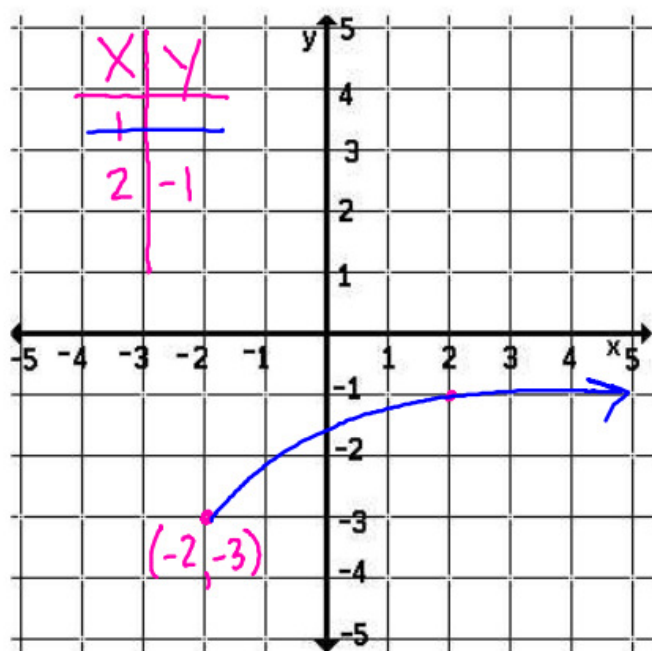


2.

$$y = \sqrt{x}$$

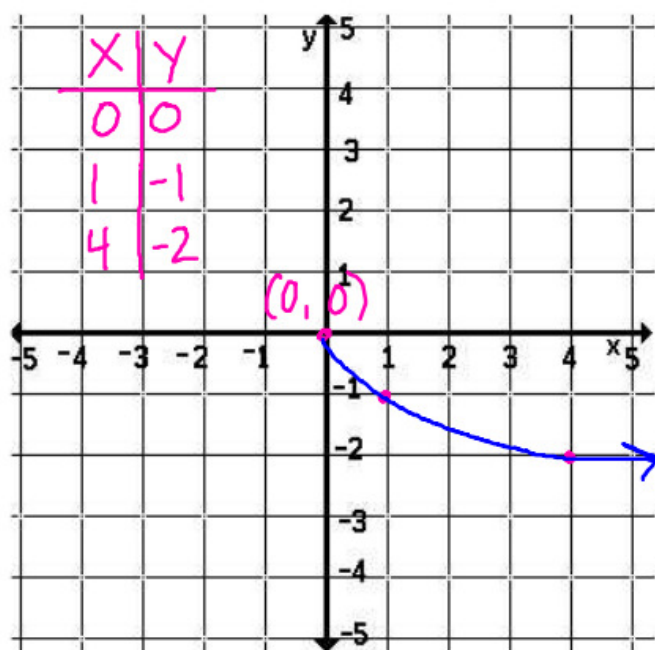


$$y = \sqrt{x+2} - 3$$

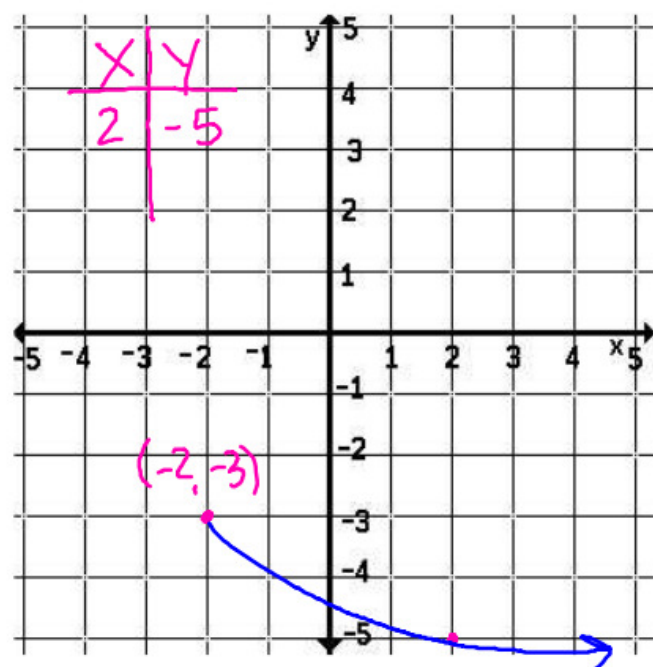


3.

$$y = -\sqrt{x}$$



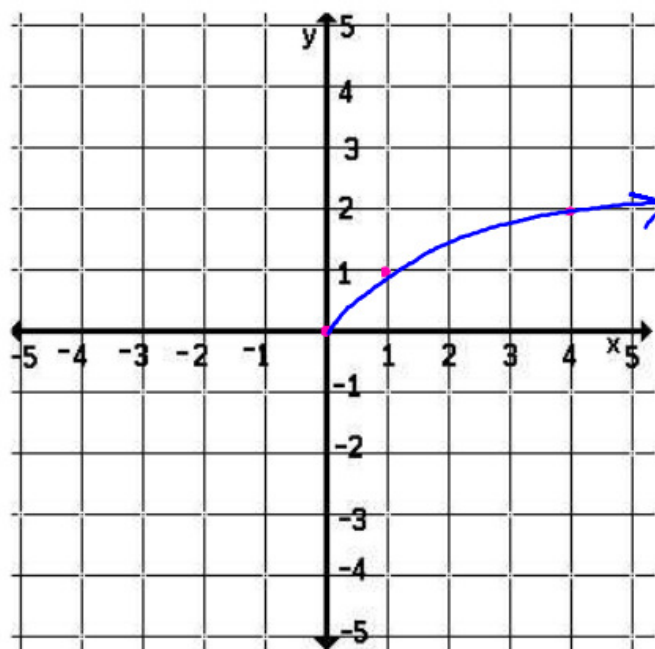
$$y = -\sqrt{x+2} - 3$$



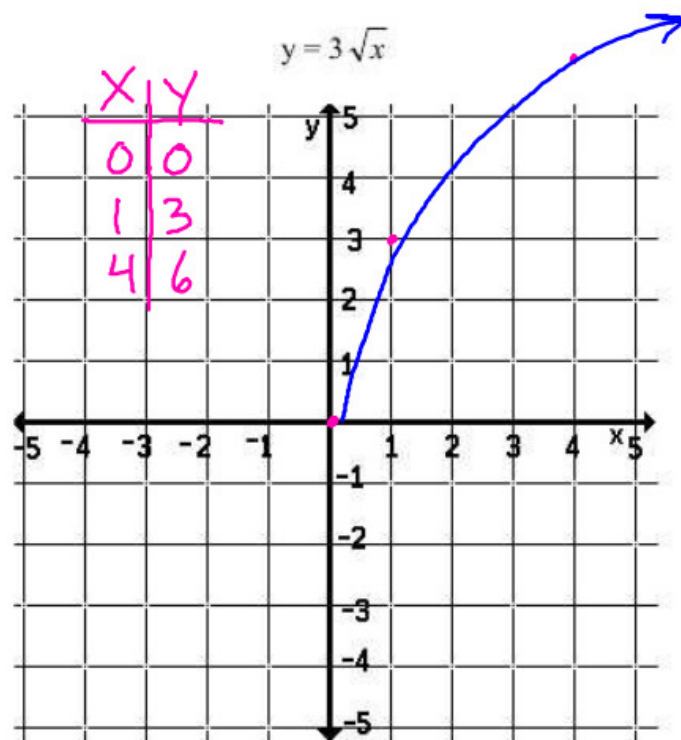
$$y = \pm\sqrt{x} \hookrightarrow$$

4.

$$y = \sqrt{x}$$

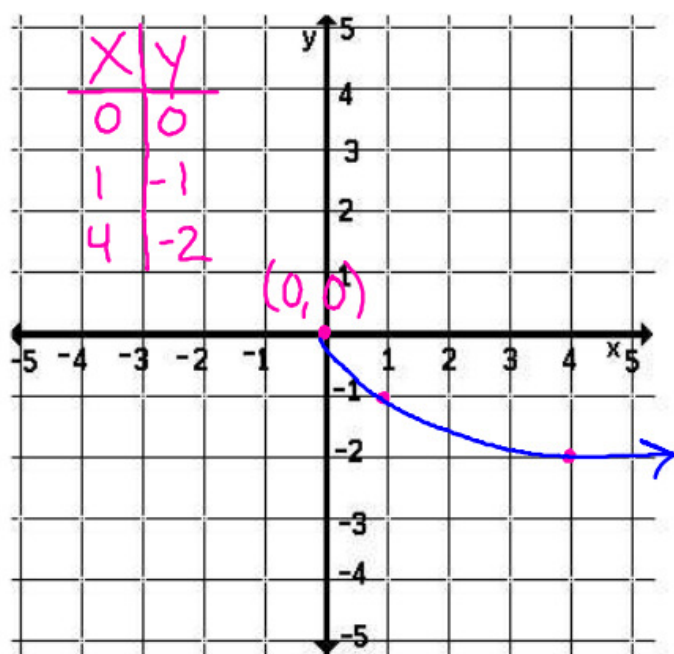


$$y = 3\sqrt{x}$$

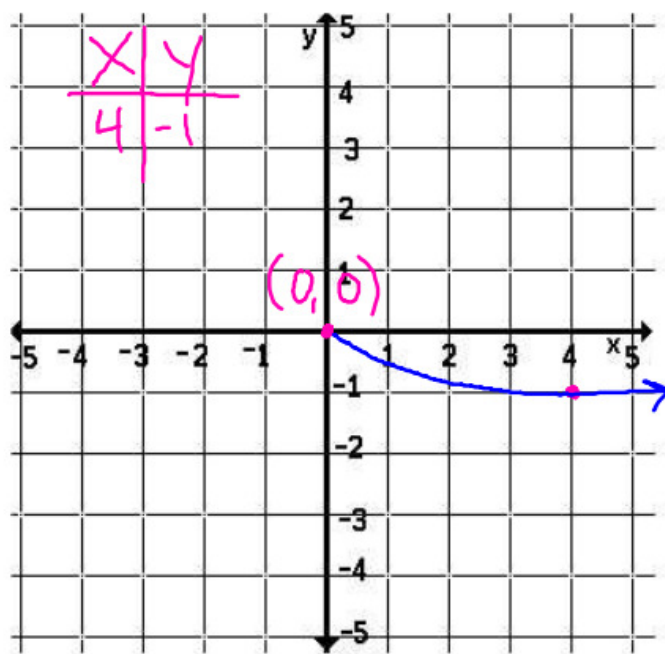


5.

$$y = -\sqrt{x}$$

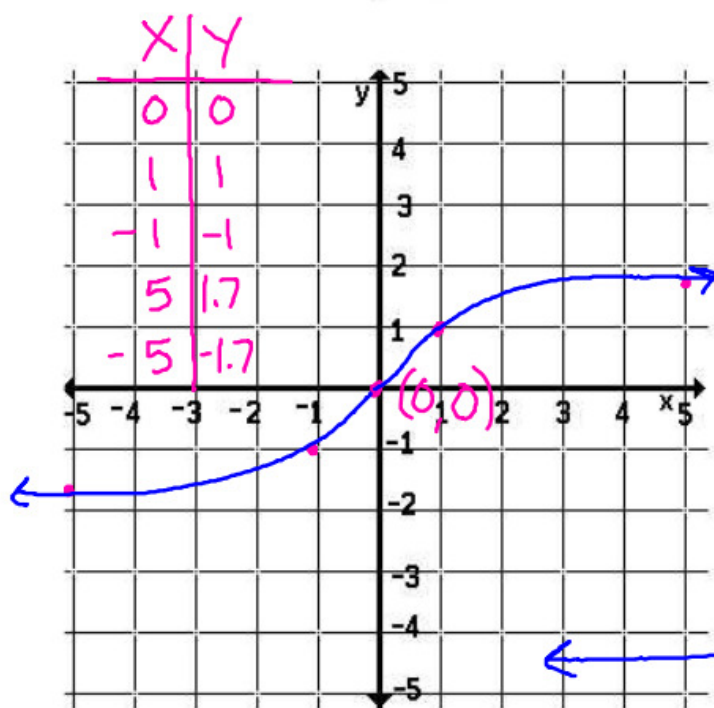


$$y = -\frac{1}{2}\sqrt{x}$$



6.

$$y = \sqrt[3]{x}$$



$$y = \sqrt[3]{x+2} - 3$$

