Sec 2-3 and 9-1

Statement	Equation
y varies directly as x	$y = kx$ ↑ Constant of variation
y varies directly as x^2	$y = kx^2$
y varies jointly with x and z	$y = kxz$

Statement	Equation
y varies inversely as x	$y = \frac{k}{x}$
y varies directly as x and inversely as z	$y = \frac{kx}{z}$

Ex y varies directly as x. If y = 4 when x = -2, find x when y = 16.

$y = kx$

$4 = k(-2)$

$-2 = k$

$y = -2x$

$16 = -2x$

$-8 = x$

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{-2}{4} = \frac{x_2}{16}$$

$$4x_2 = -32$$

$$x_2 = -8$$

Ex x and y vary inversely. If $x=5$ when $y=1.6$, find y when $x=16$

$y = \frac{k}{x}$

$1.6 = \frac{k}{5}$

$8 = k$

$y = \frac{8}{x}$

$y = \frac{8}{16}$

$y = \frac{1}{2}$

$x_1 y_1 = x_2 y_2$

$(5)(1.6) = (16) y_2$

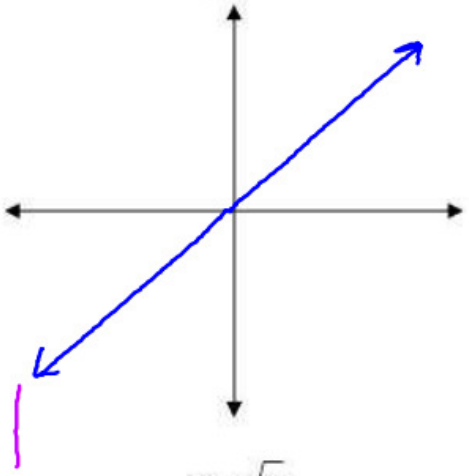
$8 = 16 y_2$

$\frac{1}{2} = \frac{8}{16} = y_2$

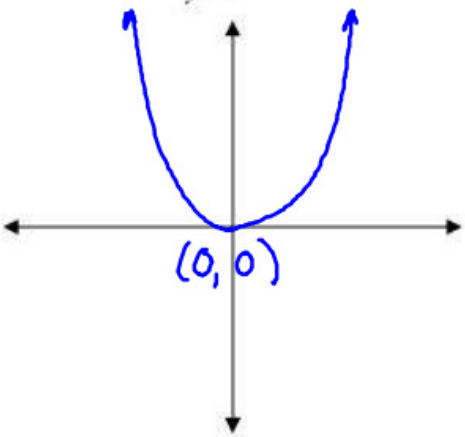
Sec 9-2

GRAPHS

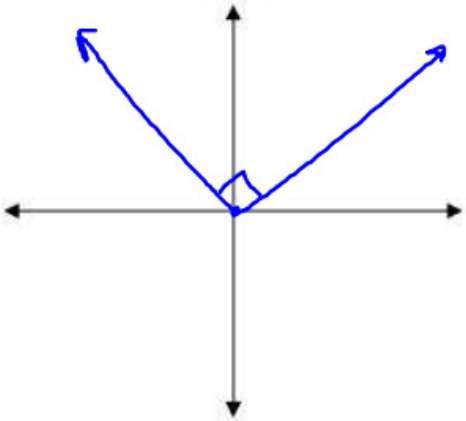
$y = x$



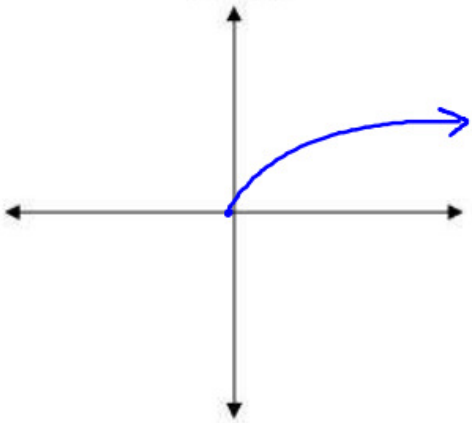
$y = x^2$



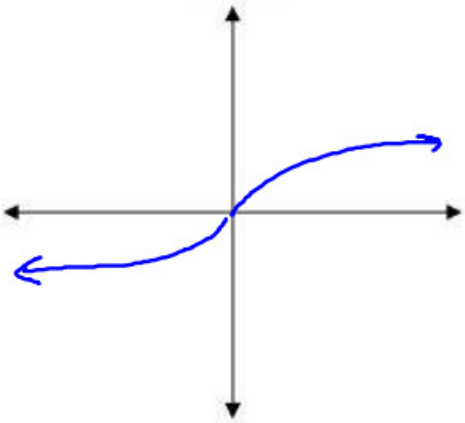
$y = |x|$



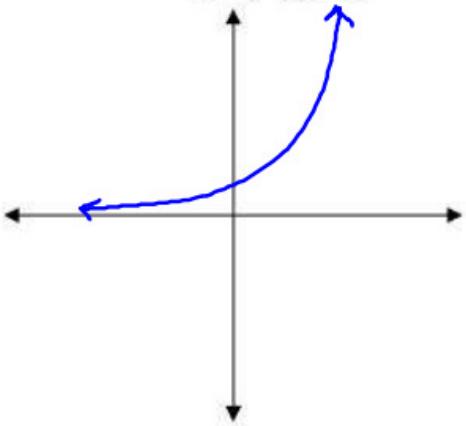
$y = \sqrt{x}$



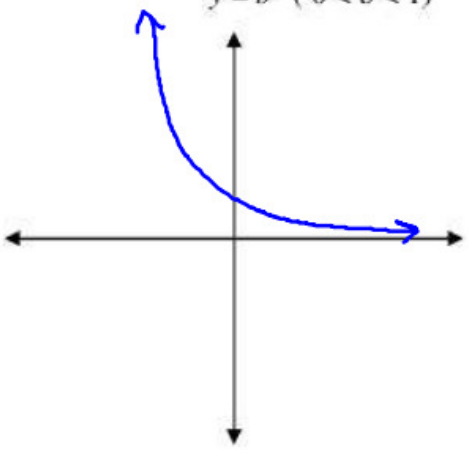
$y = \sqrt[3]{x}$



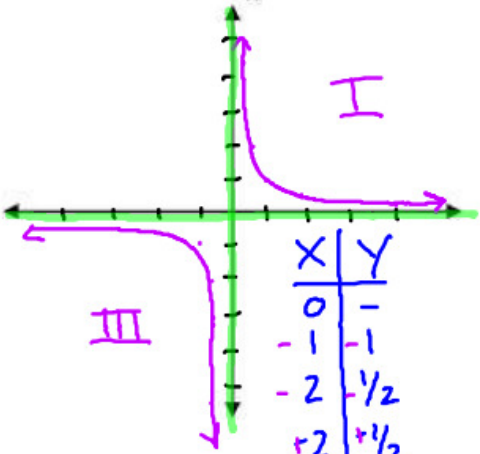
$y = b^x \ (b > 1)$



$y = b^x \ (0 < b < 1)$

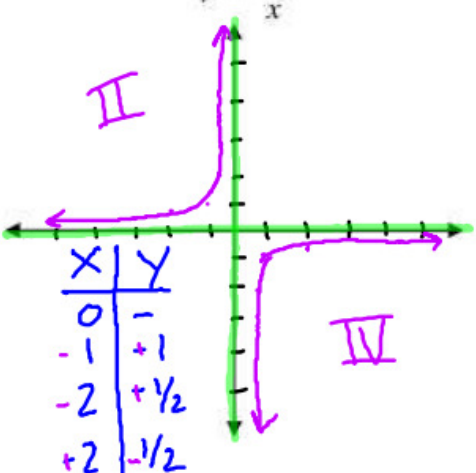


$y = \frac{1}{x}$



X	Y
0	–
-1	-1
-2	-1/2
+2	+1/2
-1/2	-2
+1/2	+2

$y = -\frac{1}{x}$



X	Y
0	–
-1	+1
-2	+1/2
+2	-1/2
-1/2	+2
+1/2	-2

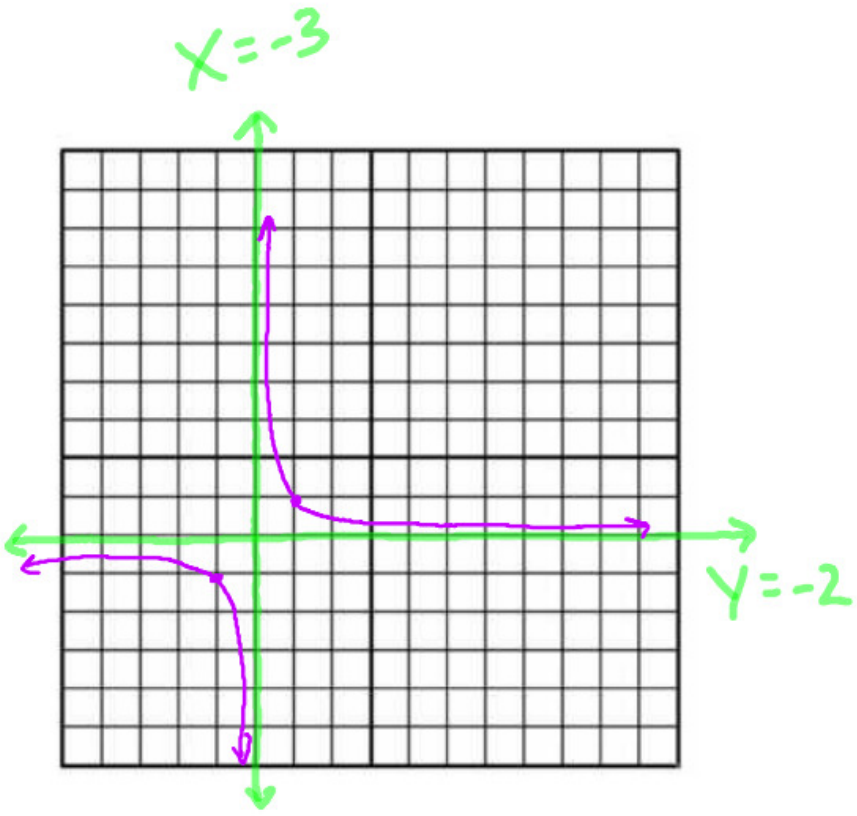
Translations of Inverse Variations

$$y = \frac{a}{x-h} + k$$

$$y = \frac{1}{x+3} - 2$$

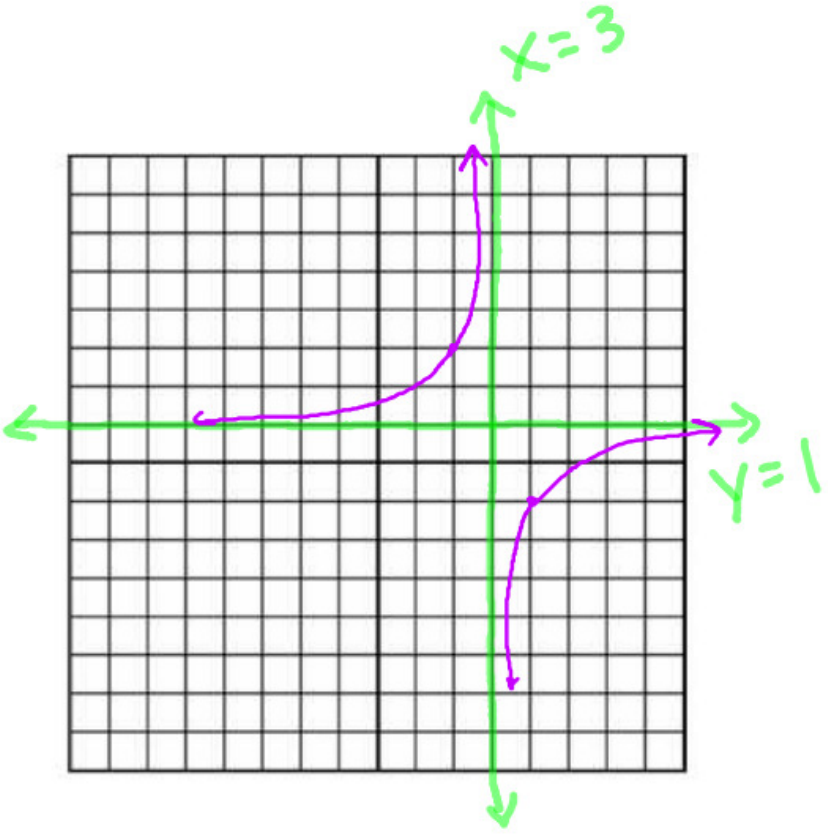
X	Y
-2	-1
-4	-3

↑ Denom = ±1



$$y = -\frac{2}{x-3} + 1$$

X	Y
2	3
3	-1



Sec 9-3

GRAPHING RATIONAL FUNCTIONS

$$f(x) = \frac{P(x)}{Q(x)}$$

1. **a.** Determine vertical asymptotes and points of discontinuity (if any) using each real zero of the denominator $Q(x)$

b. Graph and label

2. **a.** Determine horizontal asymptote (if any) using degrees of the numerator $P(x)$ and denominator $Q(x)$:

I.	Degree of $P(x)$ < Degree of $Q(x)$	$y = 0$
II.	Degree of $P(x)$ > Degree of $Q(x)$	None
III.	Degree of $P(x)$ = Degree of $Q(x)$	$y = \frac{a}{b}$

b. Graph and label

3. **a.** Calculate y-values near vertical asymptotes

b. Finish graphing

$$y = \frac{x+3}{x^2-6x+5}$$

$(x-5)(x-1)$

1. Vertical asymptotes/~~holes of discontinuity~~:

$x = 5$ and $x = 1$

2. Horizontal asymptote:

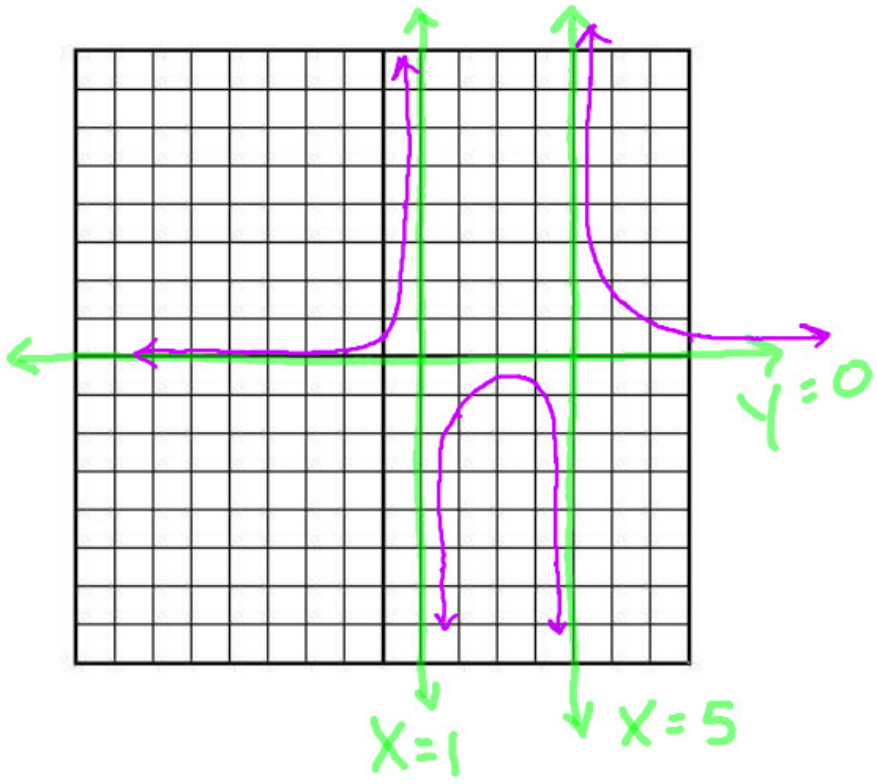
$y = 0$ (Num < Denom)

$$y = \frac{x+3}{x^2-6x+5}$$

X	Y
0	.6
-1	.2
$\frac{1}{2}$	4

X	Y
5.1	19.8
6	1.8
8	.5

X	Y
1.2	-5.5
2	-1.7
4.6	-5.3



$$y = \frac{4x-1}{x+2}$$

1. Vertical asymptotes/~~holes of discontinuity~~:

$X = -2$

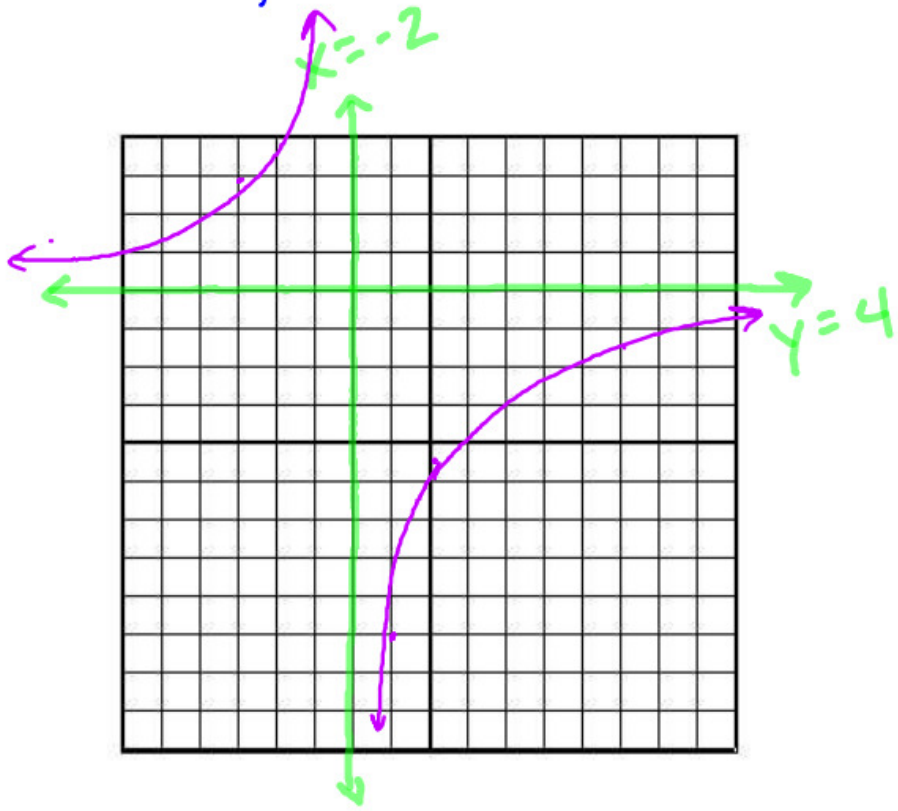
2. Horizontal asymptote:

$Y = 4$ (Num = Denom)

$$y = \frac{4x-1}{x+2}$$

X	Y
-3	13
-5	7
-10	5 1/8

X	Y
-1	-5
0	-1/2
5	2.7



$X(X+2)$

$$y = \frac{x^2 + 2x}{x+2}$$

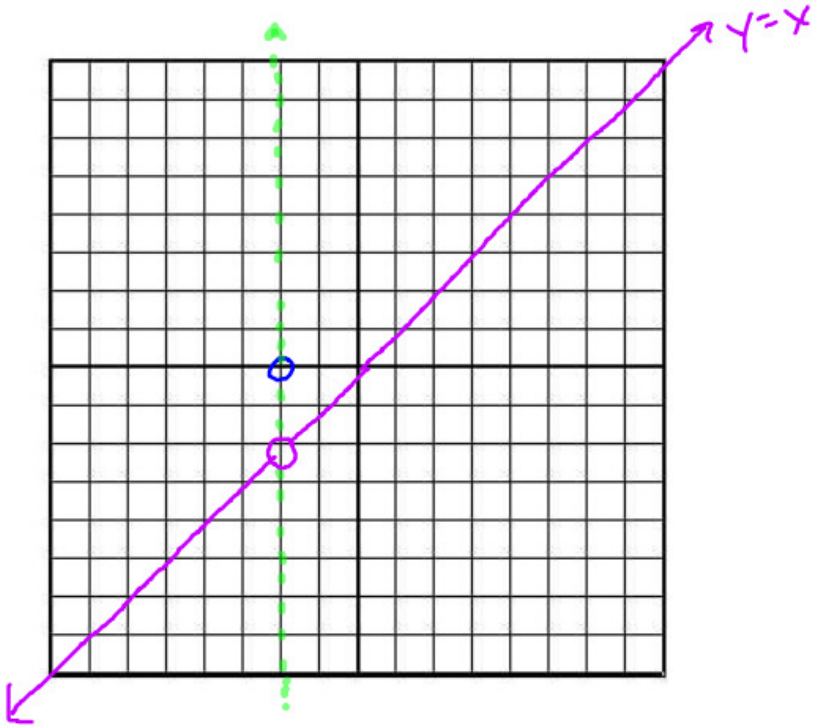
1. Vertical asymptotes/holes of discontinuity:

Hole at $X = -2$

2. Horizontal asymptote:

None (Num > Denom)

$y = x$



$$y = \frac{1}{x^2 + x - 6}$$

$(x+3)(x-2)$

1. Vertical asymptotes/holes of discontinuity:

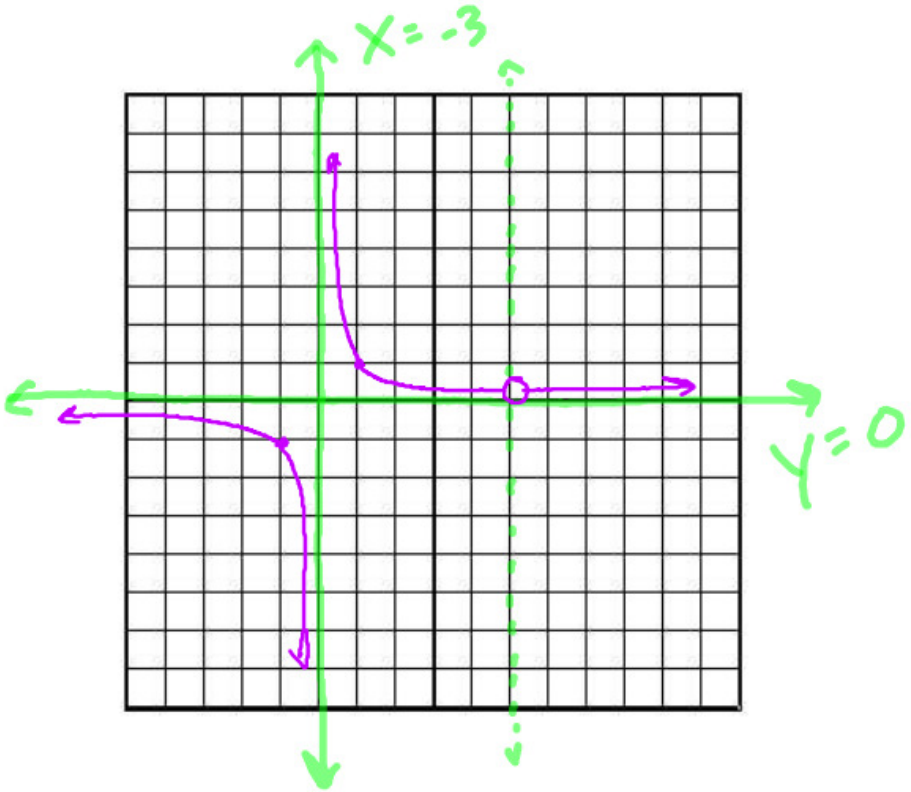
$x = -3$ Hole at $x = 2$

2. Horizontal asymptote:

$y = 0$ (Num < Denom)

$$y = \frac{1}{x+3}$$

X	Y
-2	1
-4	-1



Answer Sheet → Asymptotes