

Sec 9-4

FACTORING TECHNIQUES (Review)

★ COMMON FACTORS

$$4x^5 + 12x^4 - 8x^3 = 4x^3(x^2 + 3x - 2)$$

GROUPING

$$12x^2 + 8ax - 15x - 10a = (12x^2 + 8ax) - (15x + 10a)$$

$$= 4x(3x + 2a) - 5(3x + 2a) = (3x + 2a)(4x - 5)$$

QUADRATIC TRINOMIALS (Discriminate Test)

$$x^2 + 4x - 12 = (x + 6)(x - 2)$$

$$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$$

} FOIL

DIFFERENCE OF SQUARES

$$4x^2 - 25 = (2x - 5)(2x + 5)$$

} Conjugates

DIFFERENCE OF CUBES

$$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$$

SUM OF CUBES

$$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$$

HIGHER DEGREE POLYNOMIALS (WHICH WON'T GROUP)

$$3x^3 - 2x^2 - 7x - 2 = \frac{b}{a} = \frac{\pm 1 \pm 2}{\pm 1 \pm 3} = \pm 1 \pm \frac{1}{3} \pm 2 \pm \frac{2}{3}$$

$$\begin{array}{r} -1 \overline{) 3 \quad -2 \quad -7 \quad -2} \\ \underline{3 \quad -5 \quad -2} \quad \text{⓪} \end{array} \rightarrow (x + 1)(3x^2 - 5x - 2) = (x + 1)(3x + 1)(x - 2)$$

Multiplying Rational Expressions (Fractions)

Ex

$$\frac{15}{16} \cdot \frac{8}{9}$$
$$= \frac{\overset{1}{\cancel{(3)}}(5)}{\underset{\underset{1}{|}}{\underset{\underset{1}{|}}{\underset{\underset{1}{|}}{\cancel{(2)}\cancel{(2)}\cancel{(2)}\cancel{(2)}}}}}} \cdot \frac{\overset{1}{\cancel{(2)}}\overset{1}{\cancel{(2)}}\overset{1}{\cancel{(2)}}}{\underset{\underset{1}{|}}{\cancel{(3)}}(3)}}$$
$$= \frac{5}{6}$$

} Factor & Cancel

Ex $\frac{x^2 + x - 2}{x^2 - 4x - 12} \cdot \frac{x^2 - 5x - 6}{x^2 - 2x + 1}$

$= \frac{\frac{(x+2)(\cancel{x-1})}{(\cancel{x-6})(\cancel{x+2})} \cdot \frac{(\cancel{x-6})(x+1)}{(\cancel{x-1})(x-1)}}$

} Factor & Cancel

$= \frac{x+1}{x-1} \text{ BUT } x \neq 6, -2, 1$

} State Restrictions

Dividing Rational Expressions

$$\begin{aligned} \underline{\text{Ex}} \quad & \frac{x^3 + 8}{x - 2} \div \frac{x^2 - 2x + 4}{x^2 - 4x + 4} \\ &= \frac{(x+2)(\cancel{x^2 - 2x + 4})}{\cancel{x - 2}} \cdot \frac{(\cancel{x - 2})(x - 2)}{\cancel{x^2 - 2x + 4}} \end{aligned}$$

$$= (x - 2)(x + 2)$$

$$= x^2 - 4 \quad x \neq 2$$

Notes

$$\frac{X + \overset{1}{2}}{2 + X} > \text{same}$$

$$\frac{X - \overset{-1}{2}}{2 - X} > \text{opposites}$$

$$\frac{X - 2}{X + 2} > \text{conjugates}$$

$$9-5$$

Adding/Subtracting Rational Expressions

$$\underline{\text{Ex}} \quad \frac{3}{8} + \frac{5}{12} = \frac{\cancel{8}}{\cancel{20}}$$

$$= \frac{3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} + \frac{5 \cdot 2}{2 \cdot 2 \cdot 3 \cdot 2} \quad \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3$$

$$= \frac{9}{24} + \frac{10}{24}$$

$$= \frac{19}{24}$$

Ex $\frac{(7x-1)(x-2)}{\cancel{x^2} \cancel{2x} \cancel{3}} + \frac{-6x(x-3)}{\cancel{x^2} \cancel{x} \cancel{2}} \quad \left. \vphantom{\frac{(7x-1)(x-2)}{\cancel{x^2} \cancel{2x} \cancel{3}} + \frac{-6x(x-3)}{\cancel{x^2} \cancel{x} \cancel{2}}} \right\} \text{LCD} = (x-3)(x+1)(x-2)$

$(x-3)(x+1)(x-2) \quad (x-2)(x+1)(x-3)$

$$= \frac{7x^2 - 15x + 2 - 6x^2 + 18x}{(x-3)(x+1)(x-2)}$$

$$= \frac{x^2 + 3x + 2}{(x-3)(x+1)(x-2)}$$

$$= \frac{(x+2)(\cancel{x+1})}{(x-3)(\cancel{x+1})(x-2)} = \boxed{\frac{x+2}{x^2 - 5x + 6}}$$

Ex $\frac{3x}{\cancel{x^2-9}} + \frac{3 \overset{(-1)}{\cancel{3-x}} \overset{(-1)}{}}{(x-3)}$ > Trick ü

$$\frac{3x}{(x-3)(x+3)} + \frac{3}{(x-3)}$$

$$= \frac{3x}{(x-3)(x+3)} + \frac{-3 \overset{(x+3)}{}}{(x-3) \overset{(x+3)}{}}$$

$$= \frac{3x - 3x - 9}{(x-3)(x+3)}$$

$$= \frac{-9}{x^2-9} \text{ or } -\frac{9}{x^2-9} \text{ or } \frac{9}{9-x^2}$$

Ex
$$\frac{X - 5 + \frac{6}{X}}{1 - \frac{9}{X^2}}$$

$LCD = X^2$

$$\frac{X^2 \left(X - 5 + \frac{6}{X} \right)}{X^2 \left(1 - \frac{9}{X^2} \right)} = \frac{X^3 - 5X^2 + 6X}{X^2 - 9}$$

$$= \frac{X (X - 2) (\cancel{X - 3})}{(\cancel{X - 3}) (X + 3)}$$

$$= \boxed{\frac{X^2 - 2X}{X + 3}}$$

Sec 9-6

Solve

$$\frac{5x(x+6)}{\cancel{(x-5)(x+6)}} + \frac{4(x-5)}{\cancel{(x+6)(x-5)}} = \frac{54x+5}{\cancel{x^2+x-30}} \quad \cancel{(x-5)(x+6)}$$

$$5x(x+6) + 4(x-5) = 54x + 5$$

$$5x^2 + 30x + 4x - 20 = 54x + 5$$

$$5x^2 - 20x - 25 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = \cancel{5}, -1$$

> Check Solutions!

$$\frac{(3x+2)(x+2)}{(x-1)(x+2)} + \frac{(2x-4)(x-1)}{(x+2)(x-1)} = \frac{5 \overset{(x^2+x-2)}{(x+2)(x-1)}}{1 \cdot (x+2)(x-1)}$$

$$\cancel{3x^2} + 8x + 4 + \cancel{2x^2} - 6x + 4 = \cancel{5x^2} + 5x - 10$$

$$2x + 8 = 5x - 10$$

$$18 = 3x$$

$$6 = x$$

Note

- NEVER eliminate denominators when adding / subtracting fractions!

$$\begin{aligned} & 4(5) + \frac{1}{8}(3) \\ & \cancel{4(6)} + \cancel{(8)(3)} \quad \text{NO!} \\ & = 20 + 3 \\ & = 23 \end{aligned}$$