Sec 10.1

Sample Data
(SRS and Normal)

$\downarrow$

Calculate $\bar{x}$ or $\hat{p}$

$\downarrow$

Make conclusion about population parameter

Confidence Intervals

Hypothesis Tests

Confidence Intervals

- Used to "bet" on the interval which "captures" the population parameter

- Does <u>not</u> find μ (or p)

- Precision vs Confidence

Constructing A Confidence Interval

1) Determine how confident you want to be

↓

80%, 90%, 99% ?

↓

Table

2) Follow Steps (PAIS)

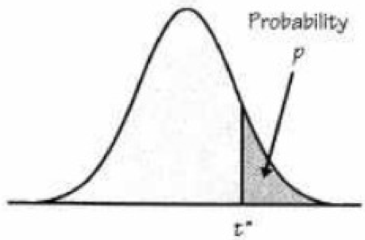


Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

TABLE C t distribution critical values

	Upper tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Critical z

P State population parameter

A State Assumptions

✓ SRS

✓ Samp Dist Normal ⇐ n is large

⇐ Pop dist normal

I Construct Interval

$$CI = \bar{X} \pm Z^* \frac{\sigma}{\sqrt{n}}$$

S State conclusion in context

CONFIDENCE INTERVALS

This test is used to calculate to a confidence interval for a population mean when ***σ is known***.

> The distribution of the systolic blood pressure of adult males is approximately normal with σ = 9.3 mmHg. Systolic readings are taken from a SRS of 27 adult males where $\bar{x}$ = 114.9 mmHg.
>
> **Construct a 99% confidence interval**
> **for the mean systolic blood pressure of all adult males.**

P) IDENTIFY POPULATION PARAMETER:

μ = avg systolic blood pressure of all adult males

A) VERIFY CONDITIONS REQUIRED FOR TEST:

✓ a) SRS - says so

✓ b) Samp Dist Normal - Pop Dist Normal

I) CONSTRUCT INTERVAL USING:

a) **TABLE C:**

i) Put data into list and calculate sample mean: ?

$\bar{X} = 114.9$

ii) Determine z* from Table C and calculate interval:

$$CI = \bar{X} \pm z^* \frac{\sigma}{\sqrt{n}} = 114.9 \pm (2.576)\frac{9.3}{\sqrt{27}} = (110.3, 119.5)$$

b) **CALCULATOR:**

$\boxed{\text{STAT}} \to$ TESTS $\to$ Z Interval = (110.29, 119.51)

S) STATE CONCLUSION:

Based on my method, I am 99% confident that the interval from 110.29 mmHg to 119.51 mmHg will capture the mean systolic blood pressure for all adult males

Ex This Wine Stinks (P.609, Ex 10.79)

DMS Odor Threshold For 10 Untrained Students:

31 31 43 36 23 34 32 30 20 24

Calculate 95% CI for mean DMS odor threshold ($\sigma = 7$ mg/L)

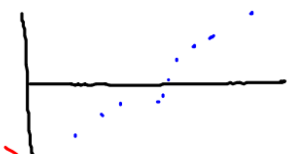
P μ = Average DMS odor threshold for all untrained students

A × SRS - Unknown ... CI may be invalid

? Sampling Dist Normal - Unknown So:

Check if data is normal

a) Check for outliers 

b) Check for normality (NPP) 

I Z Interval → Data → (26.06, 34.74)

S I am 95% confident that the mean DMS odor threshold for all untrained students is <u>between</u> 26.06 mg/L and 34.74 mg/L

Decreasing Margin of Error (MOE)

$$CI = \bar{X} \pm Z^* \frac{\sigma}{\sqrt{n}}$$

~~Decrease~~ σ

Increase Sample Size

Decrease Confidence Level (99% to 90%)

Sample Size vs MOE

Ex Surveying Hotel Managers (P.552, Ex 10.13)

114 hotel managers surveyed had spent 11.78 years on average with the company ($\sigma = 3.2$ yrs)

How large of a sample would be needed to estimate the mean within ± 1 year with 99% confidence.

$$99\% \rightarrow 2.576$$

$$CI = \bar{X} \pm \left[z^{*} \frac{\sigma}{\sqrt{n}} \le 1 \right]$$

$$2.576 \left(\frac{3.2}{\sqrt{n}} \right) \le 1$$

$$\frac{8.2432}{\sqrt{n}} \le \frac{1}{1}$$

$$8.2432 \le \sqrt{n}$$

$$67.9 \le n$$

$$n \ge 68$$

Sec 10.2

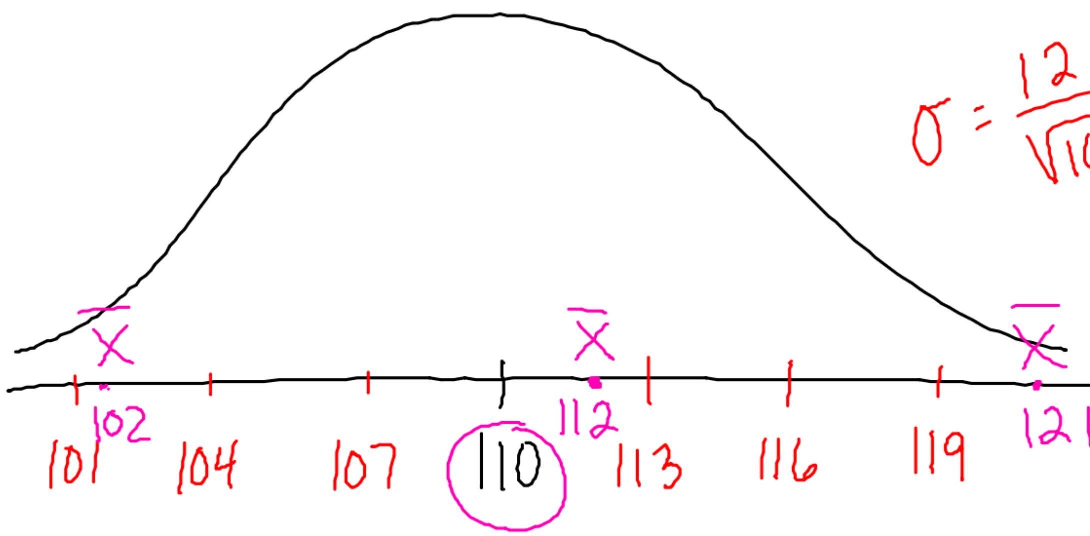
Tests of Significance (Hypothesis Tests)

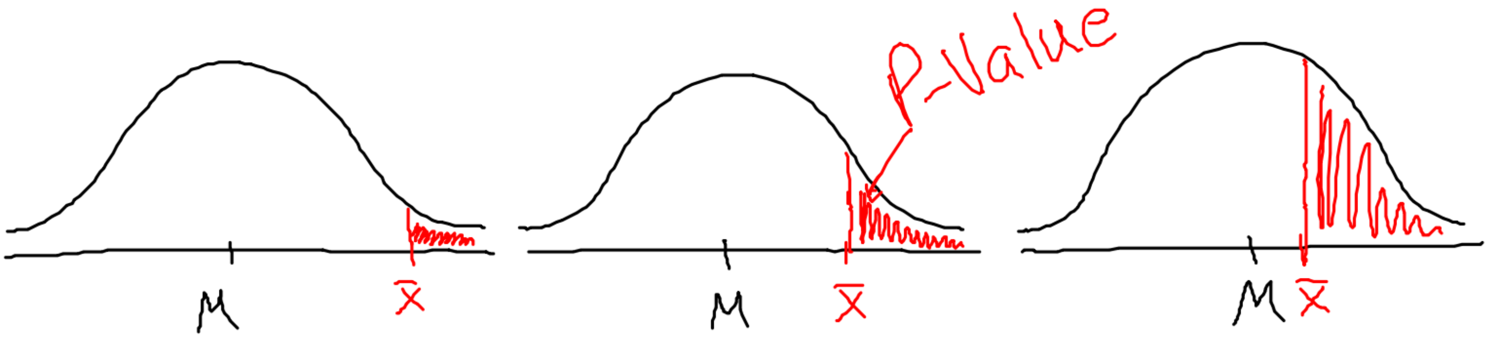
General Process

1) Guess the mean
2) Use a sample average ($\bar{x}$) to determine if guess is "reasonable"

<u>Ex</u> Principal says average IQ is 110 (where $\sigma = 12$)... SRS of 16 students.

$$\sigma = \frac{12}{\sqrt{16}} = 3$$





In General,

P-Value < 5%, statistical significance

Tests of Significance Steps (PHATS)

P State population parameter

H State "null hypothesis" and alternative/research hypothesis

$H_0: \mu = 110 \quad H_a: \mu > 110$

A State Assumptions

✓ SRS

✓ Samp Dist Nomal

T Perform test; determine P-value

S State conclusion in context

Reject Ho...

Fail To Reject Ho...

Z-TEST

This test is used to compare a sample mean ($\overline{x}$) to a population mean (μ)
when σ is known

The cellulose content of a variety of alfalfa hay is normally distributed with σ = 8mg/g.
An agronomist believes the cellulose content is higher than 140 mg/g

Test this claim at the α = .05 significance level.

α Level

To test the claim, an SRS of 15 cuttings is taken with an average cellulose content of 145 mg/g.

P) STATE POPULATION PARAMETER:

μ = average cellulose content of a variety of alfalfa hay

H) STATE HYPOTHESES:

$H_0: \mu = 140$ mg/g $\quad H_a: \mu > 140$ mg/g

A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) ✓SRS - Says so in problem

b) ✓Sampling distribution normal- normal population or large sample size [illegible] or justification for normality [illegible] after omitting outliers

Pop dist normal

T) PUT DATA INTO LIST (IF NECESSARY) AND

a) USE TABLE C:

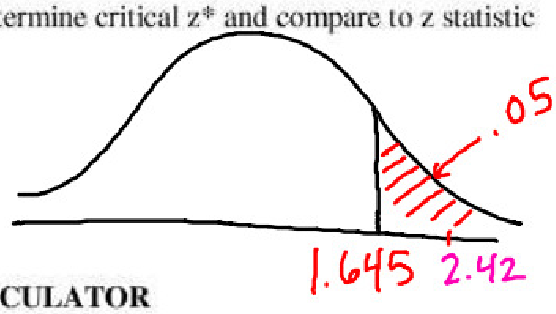
i) Determine mean ($\bar{x}$)

$$\bar{x} = 145$$

ii) Calculate z statistic

$$Z = \frac{x - \mu}{\sigma} \rightarrow \quad z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{145 - 140}{\frac{8}{\sqrt{15}}} = 2.42$$

iii) Determine critical z* and compare to z statistic



b) USE CALCULATOR

STATS ---> TESTS ---> 1: Z-Test ---> P-Value = .007

DISTR ---> 2:normalcdf (min, max) = .007

S) STATE CONCLUSION:

At $\alpha = .05$ significance level, there is sufficient evidence ($p = .007$) to reject H_0 and conclude that the mean level of cellulose content in a variety of alfalfa hay is more than 140 mg/g

CONFIDENCE INTERVAL (Use PAIS):

A 90% confidence interval for the mean cellulose content of this variety of alfalfa hay is:

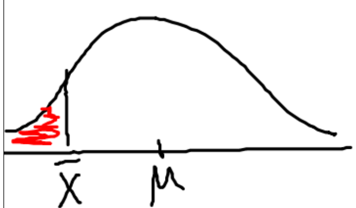
STAT ---> TESTS ---> 7:Z Interval = (141.6, 148.4)

We are 90% confident that the average cellulose content of this type of alfalfa hay is between 141.6 mg/g and 148.4 mg/g.

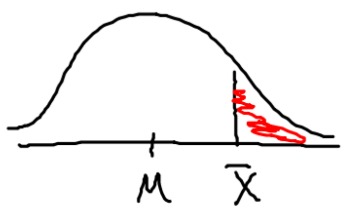
Alternative/Research Hypotheses

One Tailed

$H_a: \mu < 110$

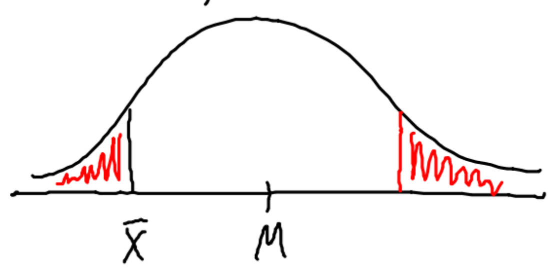


$H_a: \mu > 110$



Two Tailed

$H_a: \mu \neq 110$



Ex Has mean weight of Ft. Harrison frogs changed?

(Used to be 350 g, $\sigma = 40$ g, normal dist)

SRS of 20 frogs $\rightarrow$ $\bar{x} = 335$ g

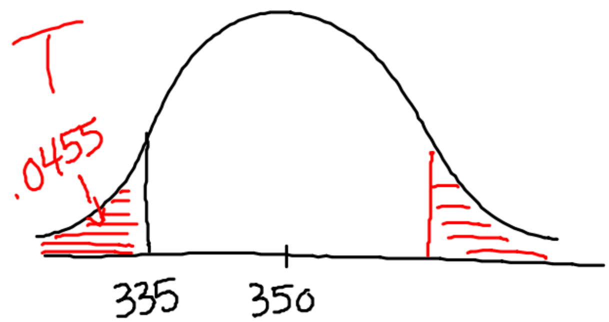
P μ = mean weight of all Ft Harrison frogs

H $H_0: \mu = 350$ g $H_a: \mu \neq 350$ g

A ✓ SRS - used

✓ Norm Samp Dist - Pop Dist Norm

T



$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{335-350}{\frac{40}{\sqrt{20}}} = -1.69$$

P-Value = 2(.0455) = .091

S At $\alpha = .05$ significance level there is insufficient evidence to reject H_0 ($P = .091$) and I must conclude that the mean weight of Ft. Harrison has not changed

Sec 10.3

Using Significance Tests Wisely

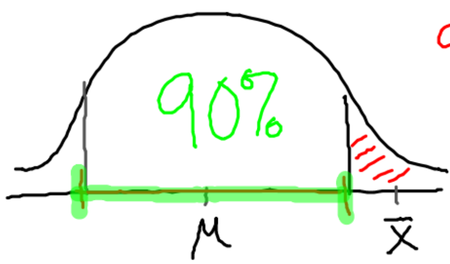
1) Verify conditions required and state implication if not met

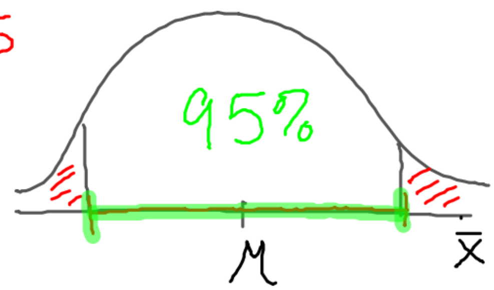
✓ SRS / Random Sample

✓ Sampling Distribution Normal

2) Reinforce results with appropriate CIs

$\alpha = .05$





3) Statistical Significance vs Practical Significance

- 2007 HSSSE

2007 HIGH SCHOOL SURVEY OF STUDENT ENGAGEMENT

Students at 100+ schools in over 30 states were asked to complete a survey…
one of the questions was:

Have you ever been bored in high school?

0	1	2	3	4
Never	Once or Twice	Once in a while	Every Day	Every Class

There was overwhelming statistical evidence ($p = .0001$)
that Lawrence North students were **less** bored in school
compared to students at the other HSSSE schools:

ACTUAL DATA:

HSSSE Mean = 2.75

LN Mean = 2.64

Sec 10.4

Significance Tests	vs	Making Decisions
- Reject H_0		- Strength of Evidence
- Fail To Reject H_0		- Many other factors

	H_0 True	H_0 False
Reject H_0 (H_0 False)	Type I Error P = α Level	Power $(1-\beta)$
Fail To Reject H_0 (H_0 True)		Type II Error P = β Level

Depends on
1) α Level
2) True value of μ

Ex

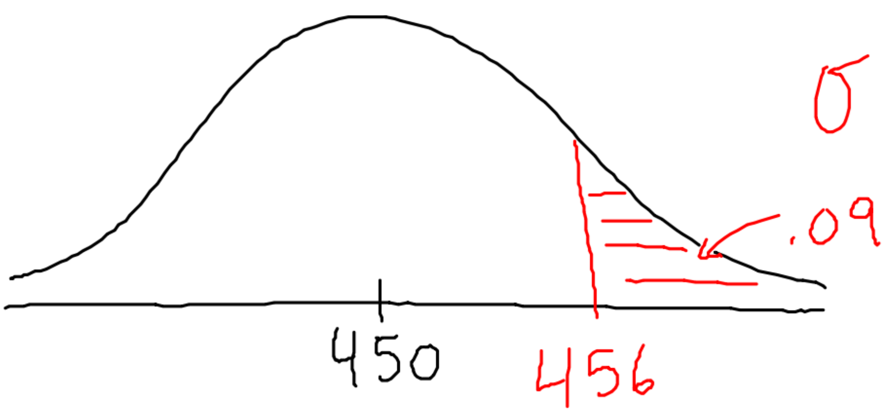
A reporter claims that if every HS senior in Indiana took the SAT then the average Math score would be no higher than 450

Hypothesis Test

SRS $(n = 500) \rightarrow \bar{x} = 456, \quad \sigma = 100$

$H_0 : \mu = 450 \qquad H_a : \mu > 450$

$$\sigma = \frac{100}{\sqrt{500}} = 4.47$$

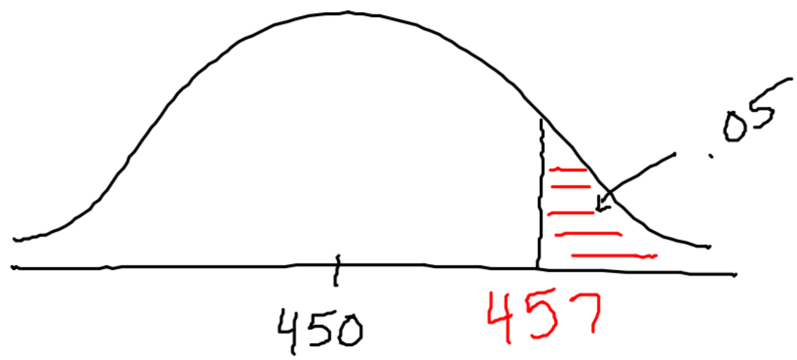


At $\alpha = .05$, we fail to reject H_0 and conclude the avg Math score is 450

- If, however, the actual mean is 460 then we have a Type II error

- Find the probability of this error

1) Determine critical $\bar{x}$ value ($\alpha = .05$)



InvNorm(.95, 450, 4.47)

2) $P(\text{Type II Error})$

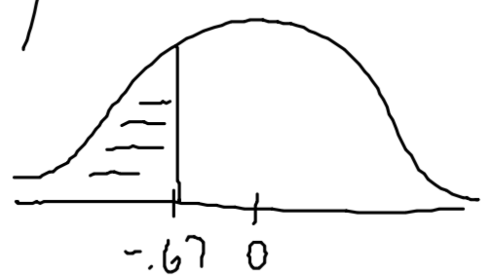
$= P(\text{"Accept" } H_0 \text{ when } \mu = 460)$

$= P(\bar{X} < 457 \text{ when } \mu = 460)$

$= P\left(Z < \dfrac{457 - 460}{4.47}\right)$

$= P(Z < -.67)$

$= .25$



If the true mean is 460 (rather than 450):

	H_0 T	H_0 F
H_0 F	$\alpha = .05$	Power = $1 - .25 = .75$
H_0 T		$\beta = .25$

If the true mean is 460 (rather than 450):

	H_0 T	H_0 F
H_0 F	$\alpha = .05$	Power = $1 - .25 = .75$
H_0 T		$\beta = .25$

At the 5% level this test will distinguish a mean of 460 from a mean of 450 in 75% of all samples

Increasing Power

1) Increase α (from 1% to 5%)

2) Increase sample size