Sec 11.1

Inference Tests

Means / Averages → t Tests

Proportions → z Tests

Categorical Data → χ^2 Tests

↑ Chi-Square (Table C)

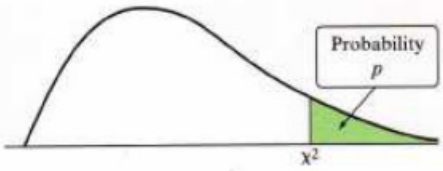


Table entry for p is the point χ^2 with probability p lying to its right.

Table C Chi-square distribution critical values

							p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Types of Chi-Square Tests

1) Goodness of Fit (GOF)

2) Homogeniety of Populations

3) Independence/Association

Steps

~~P~~ No need to state parameter(s)

H Hypotheses in words

H_0: The distributions are the same ...

H_a: The distributions are different ...

A Assumptions

1) Random sample

2) Every expected count ≥ 5

3) Independent ($N > 10n$)

T Perform Test

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

} Uses degrees of freedom

S State conclusion in context

CHI-SQUARE GOODNESS OF FIT TEST

This test is used to determine if observed counts are equal to a hypothesized distribution.

A researcher believes the Mars Company is misleading the public on its color distribution of M&Ms. He wants to compare the color distribution from a random sample of M&Ms to the Mars Company's expected values which are 10% brown, 20% red, 20% yellow, 10% green, 10% orange, 10% blue and 20% purple.

	Brown	Red	Yellow	Green	Orange	Blue	Purple
Sample	4	4	16	10	8	4	4
Expected	5	10	10	5	5	5	10

$= 50$

H STATE NULL AND ALTERNATIVE HYPOTHESES:

H_0: The color distributions of MMs are the same as the company claims

H_a: The color distributions of MMs are different than the company claims

A DETERMINE THAT CONDITIONS FOR TEST ARE ACCEPTABLE:

- ✓ Random - says so
- ✓ Every expected count ≥5 see above
- Independent $N > 50(10) > 500$ MMs in world ✓

T PERFORM TEST: } GOF-Test on calculator?

a) Calculate Chi-Square statistic:

$$\chi^2 = \frac{(4-5)^2}{5} + \frac{(4-10)^2}{10} + \frac{(16-10)^2}{10} + \frac{(10-5)^2}{5} + \frac{(8-5)^2}{5} + \frac{(4-5)^2}{5} + \frac{(4-10)^2}{10} = 18$$

b) Determine Degrees of Freedom = Number of Categories – 1 = $7-1=6$

c) Determine *P*-Value

i) Using Table C:

$P < .01$

ii) Using calculator:

DISTR → χ^2cdf $(18, 100, 6) \rightarrow P = .006$

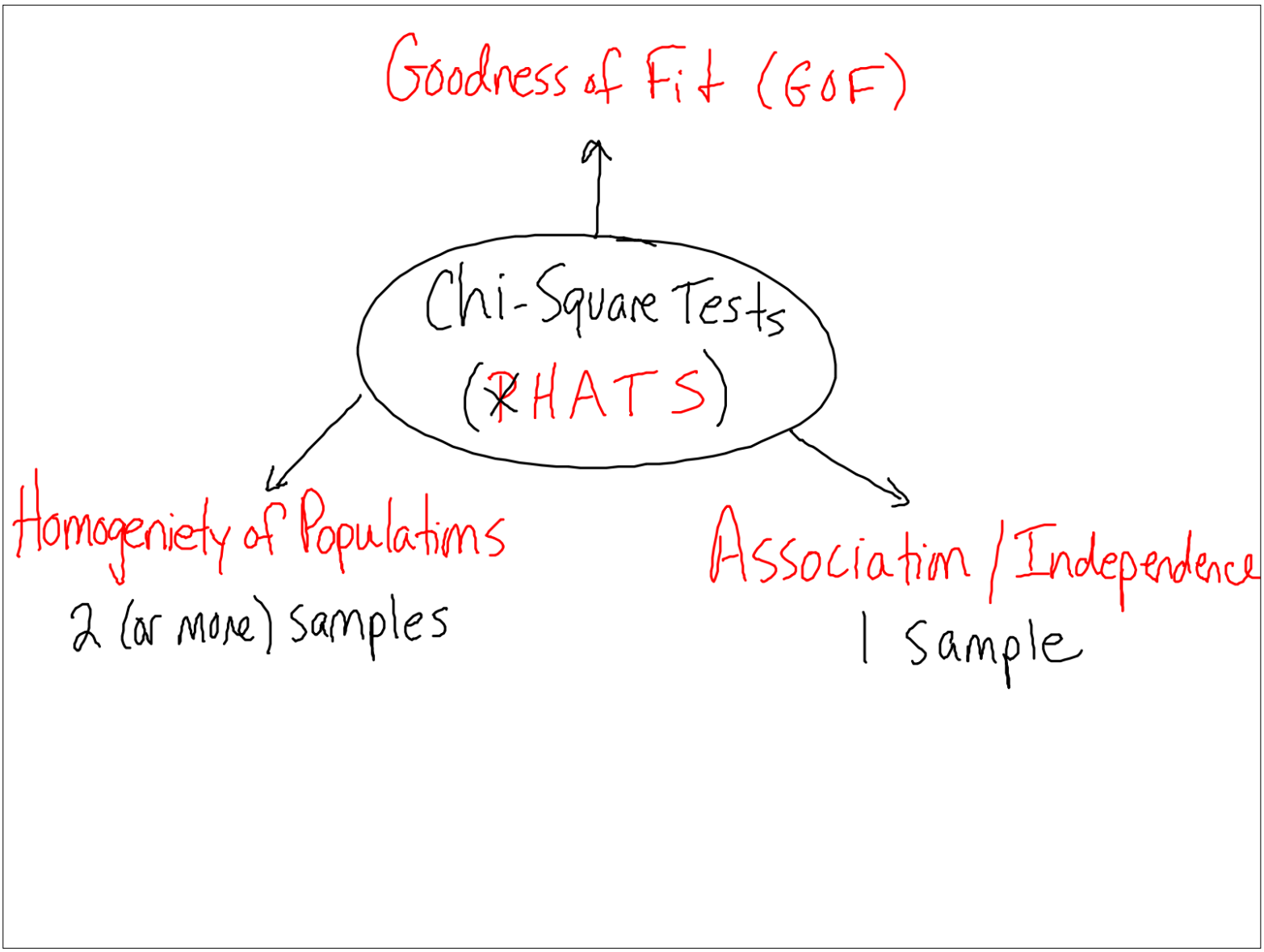
S STATE CONCLUSION IN CONTEXT

At $\alpha = .01$, there is very good evidence ($P = .006$) to reject H_0 and conclude the color distributions of M&Ms is different than what the company claims

Note

To determine which color(s) are significantly different from expected, use 2-way table techniques (Chi) and/or z-tests for proportions

Sec 11.2


Goodness of Fit (GOF)
Chi-Square Tests
(~~R~~HATS)
Homogeniety of Populations
2 (or more) samples
Association / Independence
1 sample

CHI-SQUARE TEST FOR HOMOGENEITY OF POPULATIONS

*This test is used to determine if a single categorical variable has the same distribution in 2 (or more) distinct populations from **2 (or more) samples**.*

To determine if there was an association between race and opinions about schools, researchers surveyed 3 randomly selected groups of parents and asked them "Are high schools in your state doing an excellent, good, fair or poor job or don't you know enough to say?".

	Black Parents	Hispanic Parents	White Parents	TOTAL
Excellent	12	34	22	68
Good	69	55	81	**205**
Fair	75	61	60	**196**
Poor	24	24	24	**72**
Don't Know	22	28	14	**64**
TOTAL	**202**	**202**	**201**	**605**

DETERMINE EXPECTED COUNTS:

Expected Count = (Row Total)(Column Total)/ Sample Size

	Black Parents		Hispanic Parents		White Parents	
	Actual	Expected	Actual	Expected	Actual	Expected
Excellent	12	22.7	34	22.7	22	22.6
Good	69	68.4	55	68.4	81	68.1
Fair	75	65.4	61	65.4	60	65.1
Poor	24	24.0	24	24.0	24	23.9
Don't Know	22	21.4	28	21.4	14	21.2

H STATE NULL AND ALTERNATIVE HYPOTHESES

H_0: There is no difference between race and opinions about schools

H_a: There is a difference between race and opinions about schools

A DETERMINE THAT CONDITIONS FOR TEST ARE ACCEPTABLE:

- ✓ Random - random samples used
- ✓ Every expected count ≥ 5 See Above
- ✓ Independent $N_B > 10(202) > 2020$, $N_H > 10(202) > 2020$

$N_W > 10(201) > 2010$; Respones Independent

T **PERFORM TEST USING…**

FORMULA/TABLE C:

a) Chi-Square Statistic: $X^2 = \Sigma (O_i - E_i)^2 / E_i =$

$$\chi^2 = \frac{(12-22.7)^2}{22.7} + \frac{(69-68.5)^2}{68.5} + \dots \frac{(14-21.3)^2}{21.3} = 22.43$$

b) Degrees of Freedom = $(r-1)(c-1) = (5-1)(3-1) = 8$

Number of rows in table — Number of columns in table

c) *P*-Value

i) Table C

$P < .005$

ii) Calculator:

$\chi^2 cdf(22.43, 100, 8) = .004$

CALCULATOR:

a) Store observed counts in a [R,C] matrix: R C

MATRIX → EDIT → [A] → 5 × 3 → Enter Data

b) Perform X^2 Test:

STAT → TESTS → X^2 Test → $X^2 = 22.4, P = .004$

NOTE:

If [A] = Data then [B] = Expected Counts

S **STATE CONCLUSION IN CONTEXT:**

There is significant statistical evidence ($p = .004$) to reject H_0 and conclude there is a difference between race and opinions about schools

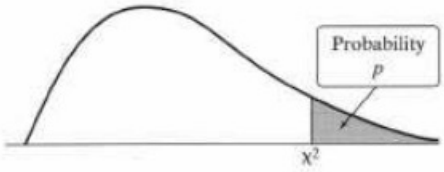


Table entry for p is the point χ^2 with probability p lying to its right.

Table C Chi-square distribution critical values

						p						
df	**.25**	**.20**	**.15**	**.10**	**.05**	**.025**	**.02**	**.01**	**.005**	**.0025**	**.001**	**.0005**
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

χ^2 Test of Association / Independence

Is there a "significant" association between 2 categorical variables from the same sample?

CHI SQUARE TEST OF ASSOCIATION/INDEPENDENCE

*This test is used to determine whether there is a significant association between 2 categorical variables from the **same sample**.*

To determine of there was a relationship between smoking status and socioeconomic levels, researchers categorized 356 federal male employees:

ACTUAL COUNTS	Socioeconomic Level			
	High	Middle	Low	
Current Smoker	51	22	43	116
Former Smoker	92	21	28	141
Never Smoked	68	9	22	99
	211	52	93	356

DETERMINE EXPECTED COUNTS:

Expected Count = (Row Total)(Column Total)/ Sample Size

EXPECTED COUNTS	Socioeconomic Level		
	High	Middle	Low
Current Smoker	68.75	16.94	30.30
Former Smoker	83.57	20.60	36.83
Never Smoked	58.68	14.46	25.86

H STATE NULL AND ALTERNATIVE HYPOTHESES

H_0: There is no association between SES and smoking in the population of federal male employees

H_a: There is an association between SES and smoking in the population of federal male employees

A DETERMINE THAT CONDITIONS FOR TEST ARE ACCEPTABLE:

? • Random - unknown; results may be invalid

✓ • Every expected count ≥ 5 See Above

? • Independent

N > 10 (356) > 3560 federal male employees

T **PERFORM TEST USING…**

FORMULA/TABLE C:

a) Chi-Square Statistic: $X^2 = \Sigma (O_i - E_i)^2 / E_i$

$$\chi^2 = \frac{(51-68.75)^2}{68.75} + \cdots \frac{(22-25.86)^2}{25.86} = 18.51$$

b) Degrees of Freedom = $(r-1)(c-1) = (3-1)(3-1) = 4$

(r – 1): Number of rows in table; (c – 1): Number of columns in table

c) *P*-Value

i) Table C:

$P < .001$

ii) Calculator:

$\chi^2 cdf(18.51, 100, 4) = .00098$

CALCULATOR:

a) Store observed counts in a [R,C] matrix:

MATRIX → EDIT → [A] → 3×3 → Enter counts

b) Perform X^2 Test:

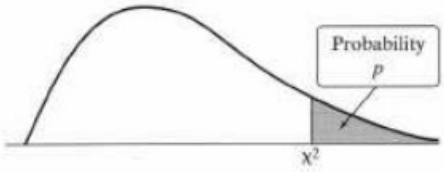
TESTS → χ^2Test → $\chi^2 = 18.51$, $p = .00098$

NOTE:

If [A]=Observed Counts; [B]=Expected Counts

S **STATE CONCLUSION IN CONTEXT:**

There is very strong evidence ($p = .00098$) to reject H_0 and conclude there is an association between smoking and socioeconomic status among federal male employees



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5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2