Sec 11.1
ONE SAMPLE T-TEST - Gosset (students't)

This test is used to compare a sample mean ($\bar{x}$) to a population mean ($\mu$) or to determine a confidence interval for a population mean when $\sigma$ is unknown.

Researchers believe that women (18-24) get less than the RDA of calcium (1200mg/day).

To test this hypothesis at the $\alpha = .05$ significance level, an SRS of 38 women between the ages of 18 and 24 years estimated their daily intakes of calcium (in mg):

<table>
<thead>
<tr>
<th>808</th>
<th>882</th>
<th>1062</th>
<th>970</th>
<th>909</th>
<th>802</th>
<th>374</th>
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</table>

P) STATE POPULATION PARAMETER:

$M = \text{avg daily intake of calcium for women 18-24 yo}$

H) STATE HYPOTHESES:

$H_0: M = 1200\, \text{mg}$ \quad $H_a: M < 1200\, \text{mg}$

A) VERIFY CONDITIONS REQUIRED FOR TEST:

- $\checkmark$ a) SRS - Says so

- $\checkmark$ b) Normal sampling distribution (normal population or large sample size or justification for normality after omitting outliers)

Sample size is large (CLT)
T) PUT DATA INTO LIST AND

a) USE TABLE C:

i) Determine mean ($\bar{x}$) and standard deviation ($s$)

$$\bar{x} = 926 \quad s = 427$$

ii) Calculate $t$ statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{926 - 1200}{\frac{427}{\sqrt{38}}} = -3.96$$

iii) Determine degrees of freedom

$$df = n - 1 = 38 - 1 = 37$$

iv) Determine critical $t$-value

Since $-3.96 \leq -1.697$ P-value $\leq .05$

b) USE CALCULATOR

STAT $\rightarrow$ TESTS $\rightarrow$ T-Test $\leftarrow t = -3.95$

DISTR $\rightarrow$ tcdf (-10, -3.96, 37) $\rightarrow$ p = .0002

S) STATE CONCLUSION:

There is significant evidence (p $\leq .05$) to reject $H_0$ and conclude that women (ages 18-24) do not get 1200 mg calcium/day

CONFIDENCE INTERVAL (Use PAIS):

$$CI = \bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

A 90% confidence interval for the mean daily intake in calcium can be found using:

STAT $\rightarrow$ TESTS $\rightarrow$ 8: T Interval = (809, 1043)

We are 90% confident that the average daily intake of calcium for women between the ages of 18 and 24 years old is between 809 mg and 1043 mg.
Table entry for $\tau^*$ and $C$ is the critical value $\tau^*$ with probability $\rho$ lying to its right and probability $C$ lying between $-\tau^*$ and $\tau^*$.

### Table C: $t$ Distribution Critical Values

<table>
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### Confidence Level $C$:

- 50% 60% 70% 80% 90% 95% 96% 98% 99% 99.5% 99.8% 99.9%
Experimental Designs
- Between Groups
- Within Groups / Matched Pairs

\[ M = \text{avg difference (Before - After)} \]

\[ H_0: M = 0 \quad H_a: M > 0 \]
\[ H_a: M < 0 \]
\[ H_a: M \neq 0 \]
MATCHED PAIRS T TEST

This test is used to compare the responses to a treatment in a within-groups design (i.e., does an SAT prep course improve an individual’s SAT scores?).

A listening test was administered to Spanish teachers before and after an institute designed to improve Spanish listening skills.

The maximum possible score on the test was 36:

| Sub | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Pre | 30 | 28 | 31 | 26 | 20 | 30 | 34 | 15 | 28 | 20 | 30 | 29 | 31 | 29 | 34 | 20 | 26 | 25 | 31 | 29 |
| Post| 29 | 30 | 32 | 30 | 16 | 25 | 31 | 18 | 33 | 25 | 32 | 28 | 34 | 32 | 32 | 27 | 28 | 29 | 32 | 32 |

Determine if the institute improved listening skills at the 5% significance level.

CALCULATE THE DIFFERENCES BETWEEN THE 2 TREATMENTS:

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P) STATE POPULATION PARAMETER:

\[ M = \text{avg difference of listening scores for Spanish teachers attending institute} \ (\text{Post} - \text{Pre}) \]

H) STATE HYPOTHESIS:

\[ H_0: M = 0 \quad H_a: M > 0 \]

A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) SRS - Unknown; results may be invalid

b) Normal sampling distribution- normal population or large sample size or justification for normal distribution after omitting outliers

- No outliers based on a modified box plot
- NPP appears linear \( \Rightarrow \text{samp dist normal} \)
T)  PERFORM TEST:

a)  USING TABLE C:

i)  Determine mean ($\bar{x}$) and standard deviation (s)

\[ \bar{x} = 1.45 \quad s = 3.2032 \]

ii)  Calculate t statistic

\[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{1.45 - 0}{3.2032 / \sqrt{20}} = 2.024 \]

iii)  Determine degrees of freedom

\[ df = 20 - 1 = 19 \]

iv)  Determine critical t-value; P-value

Since $2.024 > 1.729$
the P-value $\leq .05$

b)  USING CALCULATOR:

\[ \text{STAT} \rightarrow \text{TESTS} \rightarrow \text{TTest} \quad t = 2.02 \quad p = .0286 \]

S)  STATE CONCLUSION:

At $\alpha = .05$, there is evidence ($p = .0286$) to reject $H_0$ and conclude the institute improves listening skills for Spanish teachers attending it

CONFIDENCE INTERVAL (Use PAIS):

A 90% confidence interval for the mean increase in listening scores can be found using:

\[ \text{STAT} \rightarrow \text{TESTS} \rightarrow 8: \text{T Interval} = (.21, 2.69) \]

We are 90% confident that the mean increase in the listening scores was between .21 and 2.69 points after teachers participated in the institute.
Sec 11.2
2-Sample T Test

Compares 2 means from 2 independent samples
Conditions (for each sample)

✓ SRS

✓ Normal Sampling Distribution

✓ Independent Samples
Degrees of Freedom

- Use smaller of $n_1 - 1$ or $n_2 - 1$

- Actual formula on P. 659
Hypothesis Test

\[ t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \]

Confidence Interval

\[ CI = (\bar{X}_1 - \bar{X}_2) \pm t^* \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \]
Misc

STAT → TESTS ... → Pooled  \[\text{No}\] \[\text{yes}\]

To compare more than 2 means → ANOVA
2-SAMPLE T TEST

This test is used to compare 2 means from 2 separate (independent) samples.

Below are an SRS of math SAT scores of 13-year olds who took the test between 1980 and 1982:

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<tr>
<th>Group</th>
<th>n</th>
<th>x-bar</th>
<th>s</th>
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<tbody>
<tr>
<td>Males</td>
<td>883</td>
<td>416</td>
<td>87</td>
</tr>
<tr>
<td>Females</td>
<td>937</td>
<td>386</td>
<td>74</td>
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</table>

Determine if male scores are significantly higher than female scores at the $\alpha = .01$ level.

P) STATE POPULATION PARAMETERS:

$M_1 = M_M = \text{Avg Math SAT score of 13 yo males (1980-82)}$

$M_2 = M_F = \text{Avg Math SAT score of 13 yo females (1980-82)}$

H) STATE HYPOTHESES:

$H_0: M_M = M_F$

$H_a: M_M > M_F$

($H_0: M_M - M_F = 0$)  ($H_a: M_M - M_F > 0$)

A) VERIFY CONDITIONS REQUIRED FOR TEST:

a) SRS?

Both samples come from an SRS

b) Normal sampling distribution?

Both sample sizes are large (CLT)
T) PERFORM TEST USING

a) TABLE C:
   i) Put data into lists and calculate x-bar/standard deviation (if necessary)

   \[ 2 - \text{Var Stats?} \]
   \[ t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{416 - 386}{\sqrt{\frac{87^2}{883} + \frac{74^2}{937}}} = 7.90 \]

   ii) Calculate t-statistic:

   iii) Determine degrees of freedom:
   \[ n - 1 = 883 - 1 = 882 \]

iv) Locate critical t-value

   b) CALCULATOR:

   \[ \text{STAT} \rightarrow \text{TESTS} \rightarrow 2 \text{ Samp T Test} \rightarrow p < 0.01 \]

S) STATE CONCLUSION IN CONTEXT:

There is overwhelming evidence \((p < 0.0001)\) to reject \(H_0\) and conclude that 13 yo males did better on the Math SAT than 13 yo females between 1980 and 1982.

CONFIDENCE INTERVAL (Use PAIS):

A 98\% confidence level for the mean difference in SAT math scores between males and females can be found using:

\[ \text{STAT} \rightarrow \text{TESTS} \rightarrow 0: 2\text{-Samp T Int} = (21.16, 38.84) \]

We are 98\% confident that 13-year old males scored between 21 and 39 points higher on the SAT math test than 13-year old females between 1980 and 1982.