Sec 5.1

Success Criteria

- Interpret probability using The Law of Large Numbers
- Estimate probabilities using simulations

Big Idea

Behavior is unpredictable in the short run but a pattern emerges in the long run

Probability

- A number between 0 and 1 that describes the proportion of times an outcome occurs in the long run (Law of Large Numbers)

- Figure 5.1 (P. 285)

Simulations

- Used to estimate probabilities

Ex A couple wants to have children until they have a girl or until they have 4 children. What is the probability they have a girl?

Ex A couple wants to have children until they have a girl or until they have 4 children. What is the probability they have a girl?

Use a random digit table to estimate the probability

1) Assume / state probability

$P(\text{boy}) = .5 \quad P(\text{girl}) = .5$

2) Assign digits which preserve probability

0 - Boy	Even - Boy	0-4 Boy	etc
1 - Girl	Odd - Girl	5-9 Girl	

3) Simulate many repetitions using

0-4 ♂ 5-9 ♀

1B 9G

2B 2B 3B 9G

5G

0B 3B 4B 0B

5G

7G

5G

6G

2B 8G

7G

1B 3B 9G

6G

4B 0B 9G

1B 2B 5G

3B 1B 4B 2B

5G

4B 4B 8G

2B 8G

5G

3B 7G

4) Answer the question in context

In our simulation, a girl was "born" 18 out of 20 times so I estimate the probability that this couple will have a girl is .90 (actual = .938)

$\frac{18}{20}$

Misc Schemes (Random Digit Table)

	P(A)	P(B)	Scheme
1)	.13	.87	A = 01-13 B = 14-00

Misc Schemes (Random Digit Table)

	P(A)	P(B)	Scheme
2)	.75	.25	A = 01-75 B = 76-00
			A = 1-3 B = 4

Misc Schemes (Random Digit Table)

	P(A)	P(B)	Scheme
3)	.40	.60	A = 01-40 B = 41-00
			A = 1-2 B = 3-5
			A = 0-3 B = 4-9

Simulations Using Objects

- Coin
- Die
- Deck of Cards

Simulations Using Calculator

$\boxed{\text{MATH}}$ → PRB → randInt (min, max) → $\boxed{\text{ENTER}}$

For Repetitions → (min, max, n) → $\boxed{\text{STO}}$ → L_1

$\boxed{\text{APPS}}$ → ProbSim → $\boxed{\text{ENTER}}$ → ?

Sec 5.2

Success Criteria

- Define Sample Space, Mutually Exclusive and Independent Events
- Apply Complement and Addition Rules
- Use Venn Diagrams to determine probabilities

Definitions

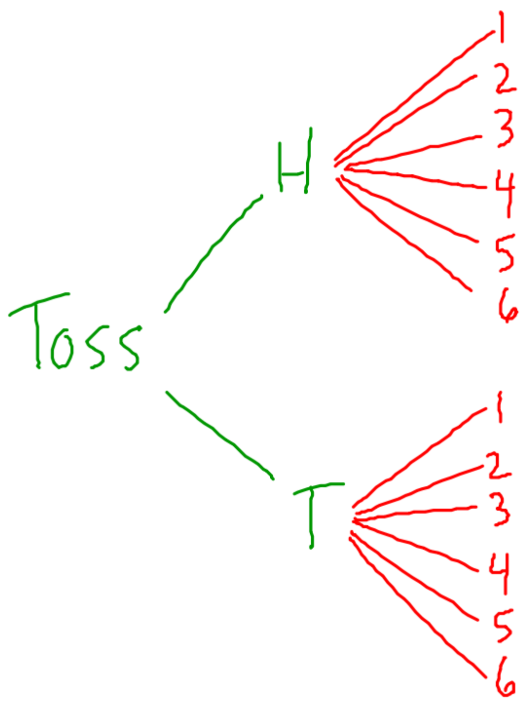
Sample Space (S)

Set of all possible outcomes of a random phenomena

Ex Letters of Alphabet

$$S = \{A, B, C, D \ldots X, Y, Z\}$$

Ex Toss A Coin $\rightarrow$ Roll a Die



$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

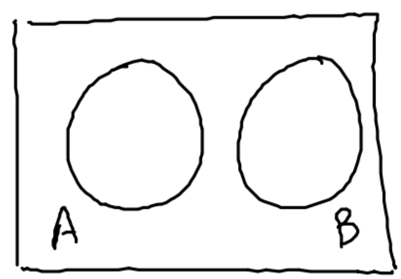
Event

- Outcome(s) of a random phenomena

- 2 Types (AP Exam)...

Mutually Exclusive (Disjoint) Events

- 2 events which cannot occur at the same time



- $P(A \text{ and } B) = 0$

- If A occurs then $P(B) = 0$

Independent Events

- Probability of one event does not affect probability of second event
- P(A) does not affect P(B)
- Knowing P(A) tells you nothing about P(B)

★ Independent events are <u>not</u> mutually exclusive!

Probability Rules (Intuitive)

1) Probability of any event occurring is between 0 and 1

$$0 \leq P(\text{Event}) \leq 1$$

2) All possible outcomes total 1

P(Heart, Diamond, Spade or Club) = 1

3) Complement Rule

$$P(A^c) = P(A') = 1 - P(A)$$

Probability that A does <u>not</u> occur

IB

4) Addition Rule

$$P(A \text{ or } B) = \underbrace{P(A) + P(B)}_{\text{Mutually Exclusive}} - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Ex In a bag of MMs, if $P(Br) = .3$ and $P(R) = .2$, find $P(Br \text{ or } R)$.

$$P(Br \text{ or } R) = P(Br) + P(R)$$

Mutually Exclusive

$$= (.3) + (.2)$$

$$= .5$$

Ex In a school, $P(\text{Math}) = .25$, $P(\text{Coach}) = .30$ and $P(\text{Both}) = .10$

Not Mutually Exclusive

Find $P(\text{Math or Coach})$:

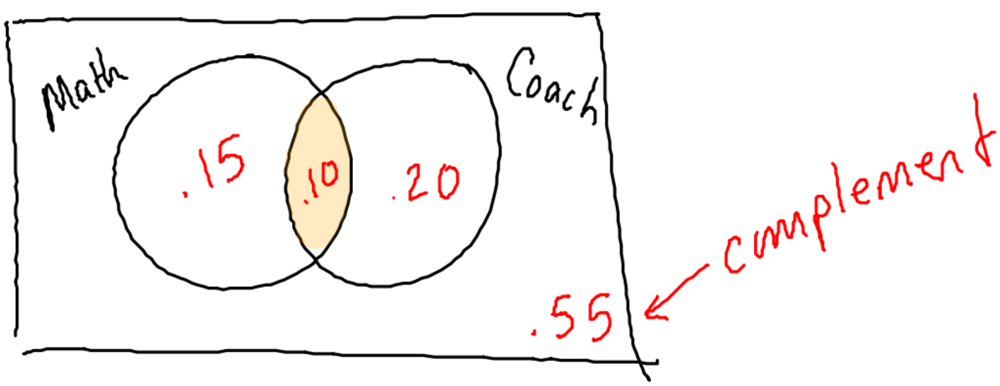
$$P(\text{Math or Coach}) = P(\text{Math}) + P(\text{Coach}) - P(\text{Math and Coach})$$

$$= (.25) + (.30) - (.10)$$

$$= .45$$

Venn Diagram

Ex In a school, 25% of teachers are math teachers, 30% are coaches and 10% are both.



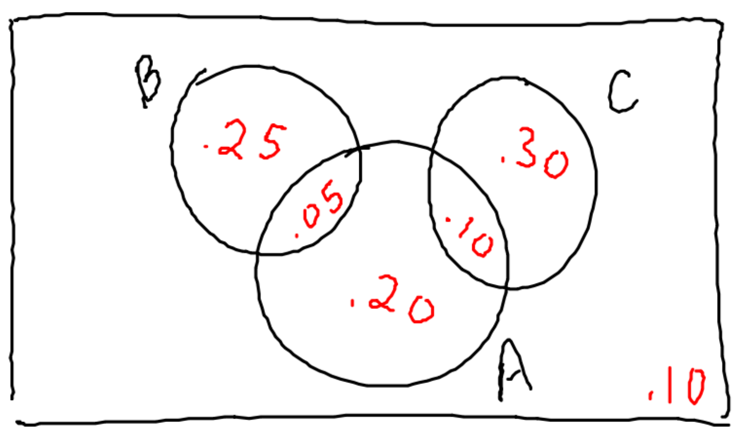
P(Math Only) = .15   P(Neither Math nor Coach) = .55

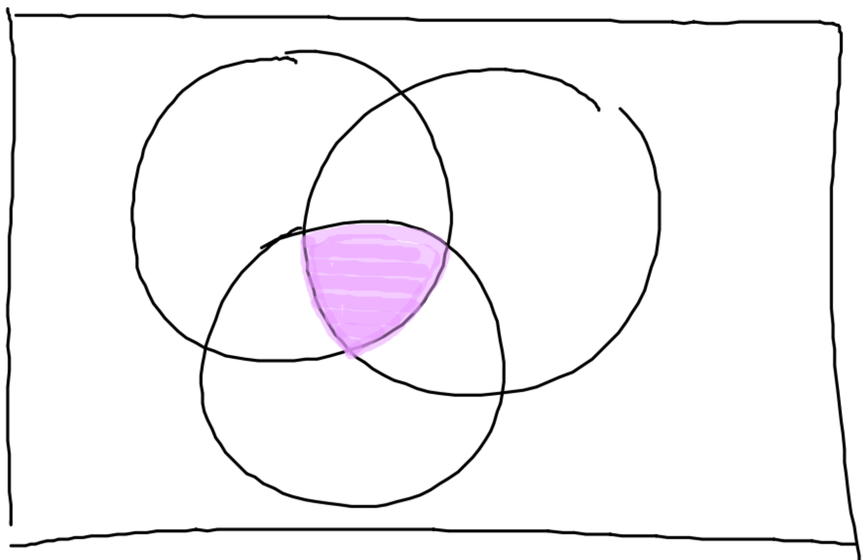
Ex Mutually Exclusive Events

$P(A) = .35 \quad P(A \cap B) = .05$

$P(B) = .30 \quad P(A \cap C) = .10$

$P(C) = .40$ B and C are mutually exclusively





If 3 events can occur at the same time, start there (shaded area)

Sec 5.3

Success Criteria

- Apply Multiplication Rule
- Calculate conditional probabilities
- Determine if 2 events are independent

5) Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$P(B$ "given that" A occured$)$

If A and B are independent then $P(B|A) = P(B)$

Ex Find P(Red and Black) with replacement:

$$P(\text{Red and Black}) = P(\text{Red}) \cdot P(\text{Black} \mid \text{Red})$$

$$= \frac{26}{52} \cdot \frac{26}{52}$$

$$= .25$$

Ex Find P(Red and Black) without replacement

$$P(\text{Red and Black}) = P(\text{Red}) \cdot P(\text{Black} \mid \text{Red})$$

$$= \frac{26}{52} \cdot \frac{26}{51}$$

$$= .2549$$

Determining Conditional Probabilities

1) Directly from problem

2) Using Formula (Probabilities Given)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \text{ and } B)}{P(B)}$$

CONDITIONAL PROBABILITY

1. Here are the counts (in thousands) of earned degrees in the United States in a recent year:

	Bachelor's	Master's	Professional	Doctorate	
Male	616	194	30	16	856
Female	529	171	44	26	770
	1145	365	74	42	1626

a. Find *P*(Randomly chosen degree recipient is a woman)

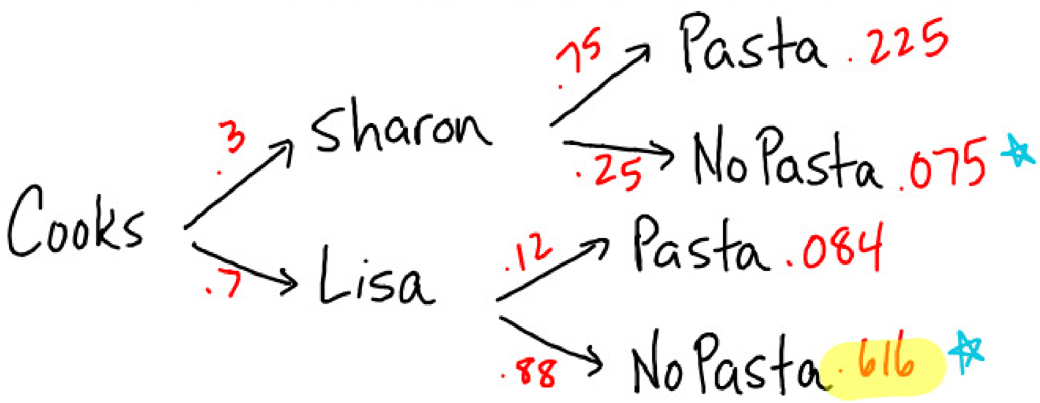
$$= \frac{770}{1626} = .4735$$

b. Find *P*(Choose a woman | She received a professional degree)

$$= \frac{44}{74} = .5945$$

2. Sharon and Lisa share an apartment. Sharon cooks dinner 3 nights out of 10. If Sharon does not cook, then Lisa does. If Sharon cooks dinner, the probability they have pasta is 0.75. If Lisa cooks dinner, the probability they have pasta is 0.12.

Find P(Lisa Cooked Dinner | They Do Not Have Pasta)



$$P(\text{Lisa Cooks} \mid \text{No Pasta}) = \frac{\text{Lisa Cooks and No Pasta}}{P(\text{No Pasta})}$$

$$= \frac{.616}{.616 + .075} = \boxed{.8915}$$

Proving Events Are Independent (or Not)

1) Show $P(A|B) = P(A)$

Ex Previous Problem . . .

Is being a woman independent of earning a professional degree?

CONDITIONAL PROBABILITY

1. Here are the counts (in thousands) of earned degrees in the United States in a recent year:

	Bachelor's	Master's	Professional	Doctorate	
Male	616	194	30	16	856
Female	529	171	44	26	770
	1145	365	74	42	1626

a. Find *P*(Randomly chosen degree recipient is a woman)

$$= \frac{770}{1626} = .4735$$

Not Independent:

b. Find *P*(Choose a woman | She received a professional degree)

$$= \frac{44}{74} = .5945$$

P(Woman) ≠ P(woman | prof degree)

2) If $P(A \text{ and } B) = P(A) \cdot P(B)$

then A and B are independent

CONDITIONAL PROBABILITY

1. Here are the counts (in thousands) of earned degrees in the United States in a recent year:

	Bachelor's	Master's	Professional	Doctorate	
Male	616	194	30	16	856
Female	529	171	44	26	770
	1145	365	74	42	1626

$$P(\text{Woman and Prof Degree}) = \frac{44}{1626}$$

$$P(\text{Woman}) = \frac{770}{1626}$$

$$P(\text{Prof Degree}) = \frac{74}{1626}$$

$$P(\text{woman and prof degree}) = P(\text{woman}) \cdot P(\text{prof degree})\,?$$

$$\frac{44}{1626} = \frac{770}{1626} \cdot \frac{74}{1626}\ ?$$

$$.0270 \neq .0215$$

$\downarrow$

Not Independent Events

Randomly pick 3 letters of the alphabet with replacement. Find the probability of getting at least one vowel.

P(At least one vowel) =

P(V and V and V) or P(V and V and C) or P(V and C and V) or P((C and V and V) or...

Trick :)

Randomly pick 3 letters of the alphabet with replacement. Find the probability of getting at least one vowel.

P(At least 1 vowel) = 1 − P(no vowels)

$$= 1 - \left(\frac{21}{26}\right)\left(\frac{21}{26}\right)\left(\frac{21}{26}\right)$$

$$= 1 - \frac{9261}{17576} = \boxed{.5269}$$

Solving Probability Problems

- Estimate With Simulations
- Intuition / Gut
- Addition / Multiplication Rules
- Tree Diagrams
- Venn Diagrams