Sec 5.1

Big Idea

Behavior is unpredictable in the short run but a pattern emerges in the long run

Probability

- A number between 0 and 1 that describes the proportion of times an outcome occurs in the long run (Law of Large Numbers)
- Figure 5.1 (P. 285)

Simulations

- Used to estimate probabilities

<u>Ex</u> A couple wants to have children until they have a girl or until they have 4 children. What is the probability they have a girl?

1) Assume / state probability

$P(\text{boy}) = .5 \quad P(\text{girl}) = .5$

2) Assign digits which preserve probability

0 - Boy	Even - Boy	0-4 Boy	etc
1 - Girl	Odd - Girl	5-9 Girl	

3) Simulate many repetitions using

a) Random Digit Table

1B	2B	5G	0B	5G	7G	5G	6G	2B	7G
9G	2B		3B					8G	
	3B		4B						
	9G		0B						

1B	6G	4B	1B	3B	5G	4B	2B	5G	3B
3B		0B	2B	1B		4B	8G		7G
9G		9G	5G	4B		8G			
				2B					

b) Calculator

$\boxed{\text{MATH}} \rightarrow$ PRB $\rightarrow$ randInt (min, max) $\rightarrow \boxed{\text{ENTER}}$

For Repetitions $\rightarrow$ (min, max, n) $\rightarrow \boxed{\text{STO}} \rightarrow$ Ln $\rightarrow$ Sort?

$\boxed{\text{APPS}} \rightarrow$ Prob Sim $\rightarrow \boxed{\text{ENTER}} \rightarrow$?

4) Answer the question in context

In our simulation, a girl was "born" 18 out of 20 times so I estimate the probability that this couple will have a girl is .90 (actual = .938)

Misc Schemes (Random Digit Table)

	P(A)	P(B)	Schemes
1)	.75	.25	A = 01-75 B = 76-00 A = 1-3 B = 4 A = 01-15 B = 16-20
2)	.40	.60	A = 01-40 B = 41-00 A = 1-4 B = 5-0
3)	.13	.87	A = 01-13 B = 14-00 A = 001-013 B = 014-100

Sec 5.2

Definitions

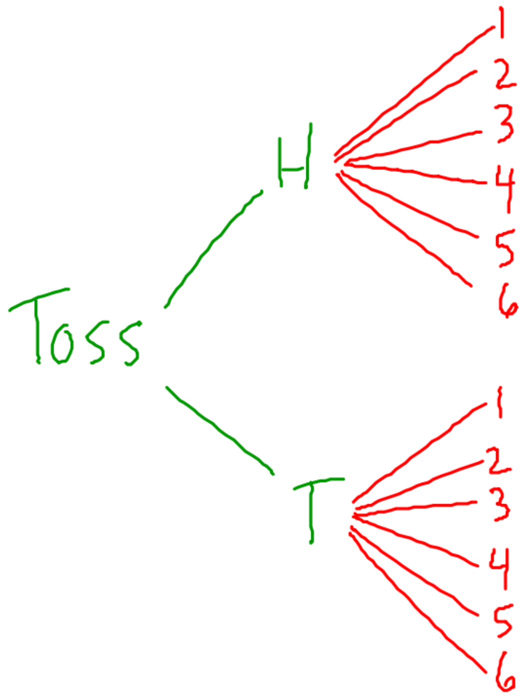
Sample Space (S)

Set of all possible outcomes of a random phenomena

Ex Letters of Alphabet

$$S = \{A, B, C, D \ldots Z\}$$

Ex Toss A Coin → Roll a Die



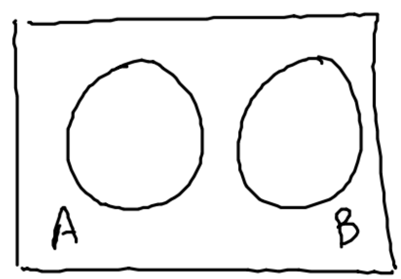
$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

Event

Outcome(s) of a random phenomena

Mutually Exclusive (Disjoint) Events

- 2 events which can<u>not</u> occur at the same time



- $P(A \text{ and } B) = 0$

- If A occurs then $P(B) = 0$

Independent Events

- Probability of one event does not affect probability of second event
- P(A) does not effect P(B)
- Knowing P(A) tells you nothing about P(B)
- Independent events are NOT mutually exclusive

Probability Rules (Intuitive)

Color	Br	R	Y	Gr	Or	Bl
Prob	.3	.2	.2	.1	.1	.1

1) Probability of any event is between 0 and 1

$$0 \leq P(\text{event}) \leq 1$$

2) All possible outcomes total 1

$P(Br, R, Y, G, Or \text{ or } Bl) = 1$

3) Complement Rule

$$P(A^c) = P(A') = 1 - P(A)$$

Probability that A does <u>not</u> occur

IB

4) Addition Rule

Mutually Exclusive

Not Mutually Exclusive

$P(A \text{ or } B) = P(A) + P(B)$

$P(A \cup B) = P(A) + P(B)$

Ex $P(Br \text{ or } R)$

$= P(Br) + P(R)$

$= .3 + .2$

$= .5$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Ex $P(\text{Math}) = .25$ $P(\text{Coach}) = .30$

$P(\text{Both}) = .10$

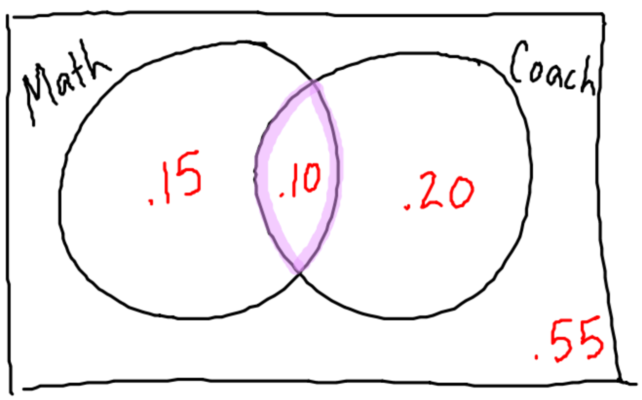
$P(\text{Math or Coach})$

$= P(\text{Math}) + P(\text{Coach}) - P(\text{Both})$

$= .25 + .30 - .10 = .45$

Venn Diagram

Ex In a school, 25% of teachers are math teachers, 30% are coaches and 10% are both.



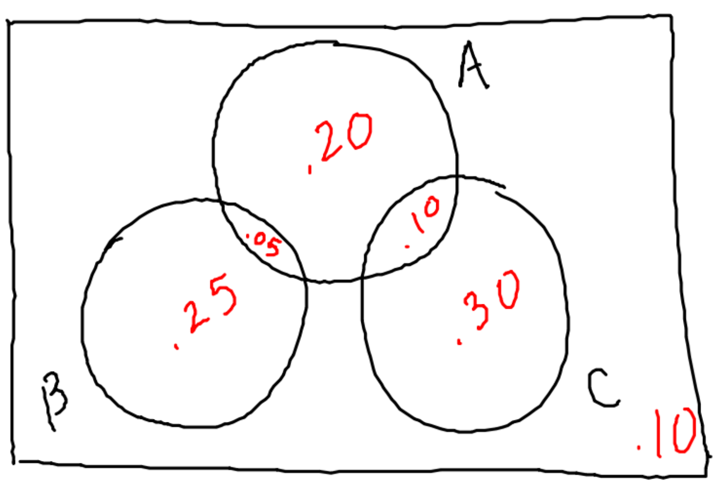
P(Math Only) = .15 P(Neither) = .55

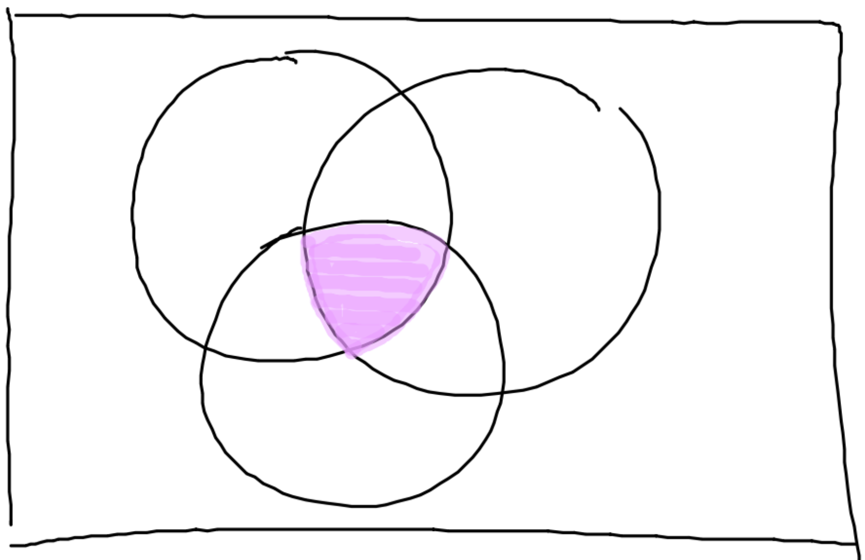
Ex Mutually Exclusive Events

$P(A) = .35 \quad P(A \cap B) = .05$

$P(B) = .30 \quad P(A \cap C) = .10$

$P(C) = .40$ B and C are mutually exclusively





If 3 events can occur at the same time,
start there (shaded area)

Sec 5.3

5) Multiplication Rule

Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex P(Red and Black) with replacement:

$= P(R) \cdot P(B)$

$= (.5)(.5)$

$= .2500$

Not Independent

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

conditional probability

Ex P(Red and Black) without replacement

$= P(R) \cdot P(B|R)$

$= \frac{26}{52} \cdot \frac{26}{51}$

$= .2549$

Determining Conditional Probabilities

1) Directly from problem

	10	11	12	
Eng	10	6	4	20
Bio	6	8	2	16
Spanish	5	4	5	14
	21	18	11	50

$P(\text{Sophomore} \mid \text{English Major}) = \frac{10}{20} = .5$

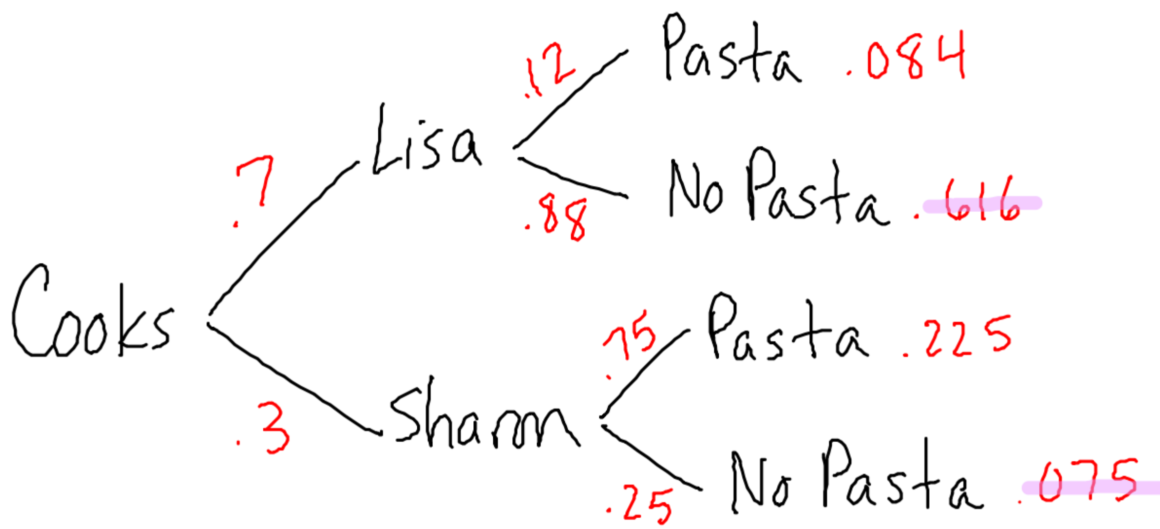
$P(\text{English} \mid \text{Sophomore}) = \frac{10}{21} = .4762$

2) Using Formula (Probabilities)

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

<u>Ex</u> Sharon and Lisa share an apartment. Sharon cooks dinner 3 nights out of 10. If Sharon does not cook dinner then Lisa does. If Sharon cooks dinner, the probability they have pasta is .75. If Lisa cooks dinner, the probability they have pasta is .12.

P (Lisa Cooked Dinner | Do Not Have Pasta)



$$P(\text{Lisa Cooks} \mid \text{No Pasta}) = \frac{P(\text{No Pasta and Lisa Cooks})}{P(\text{No Pasta})}$$

$$= \frac{.616}{.616 + .075} = .8915$$

Proving Events Are Independent (or Not)

1) Show $P(A \text{ and } B) = P(A) \cdot P(B)$

$\underline{\text{Ex}}$ $P(A) = .4$

$P(B) = .3$

$P(A \text{ and } B) = .1$

Are A and B independent?

↓

No... because

$P(A \text{ and } B) \neq P(A) \cdot P(B)$

2) Show $P(B|A) = P(B)$

<u>Ex</u> Pick 2 Cards with Replacement

$$P(\text{Red}\,|\,\text{Black}) = \frac{26}{52} = P(\text{Red})$$

Trick :)

Randomly pick 2 letters of the alphabet with replacement. Find the probability of getting at least one vowel.

$$P(\text{At least 1 vowel}) = 1 - P(\text{no vowels})$$

$$= 1 - \left(\frac{21}{26}\right)\left(\frac{21}{26}\right)$$

$$= .3476$$

Solving Probability Problems

- Intuition / Gut
- Rules
- Tree Diagrams
- Venn Diagrams
- Estimate With Simulations