Sec 6.1

Learning Targets

- Calculate mean and standard deviation of a discrete random variable
- Find probabilities of a continuous random variable

Variables

$$\triangle \rightarrow x \rightarrow \sigma$$

Random Variable (X)

A variable whose value is a probability

Discrete RVs

The probabities that a customer selects 1, 2, 3, 4 or 5 items is:

X	1	2	3	4	5
P(X)	.32	.12	.23	.18	.15

= 1.00 ✓

X = the number of items bought

$$P(X > 3.5) = P(X = 4 \text{ or } 5) = \boxed{.33}$$

$$P(1.0 < X < 3.0) = P(X = 2) = \boxed{.12}$$

$$P(X < 5) = P(X \neq 5) = 1 - .15 = \boxed{.85}$$

Mean of a RV (Weighted Average)

$$\boxed{E(X) = \mu_X = \sum x_i p_i}$$

$$\mu_X = 1(.32) + 2(.12) + 3(.23) + 4(.18) + 5(.15)$$

$$= \boxed{2.72 \text{ items}}$$

Don't Round

Variance of a RV

$$\boxed{Var(x) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i}$$

$$\sigma^2 = (1-2.72)^2(.32) + (2-2.72)^2(.12) + (3-2.72)^2(.23)$$
$$+ (4-2.72)^2(.18) + (5-2.72)^2(.15) = 2.1016$$

$$\sigma = \sqrt{2.1016} = 1.4496$$

Finding Mean / Standard Deviation Using Calculator:

L_1 = X Values

L_2 = Probabilities

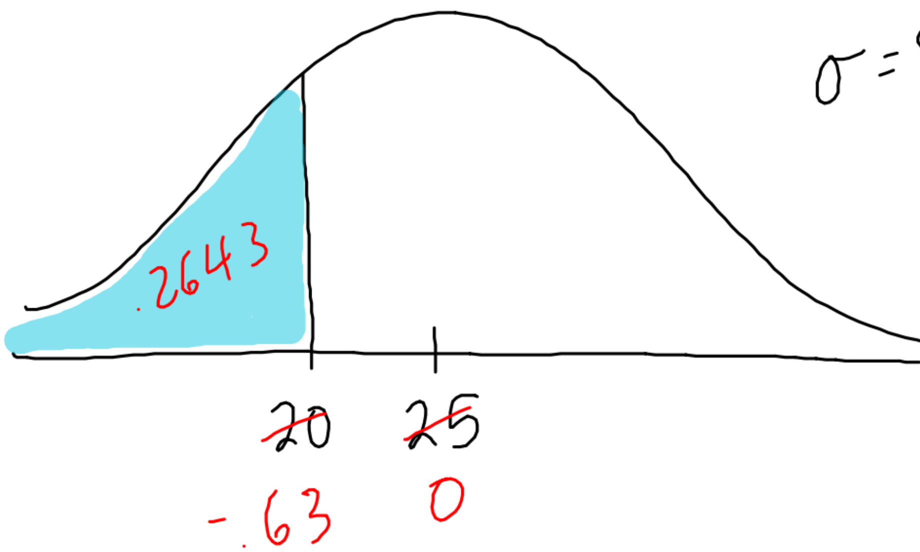
1 Var Stats $\rightarrow L_1, L_2$ (Frequency)

Continuous RVs

The time it takes Mark to do his math homework follows a Normal distribution with mean 25 minutes and standard deviation 8 minutes

Let X = time to complete math homework

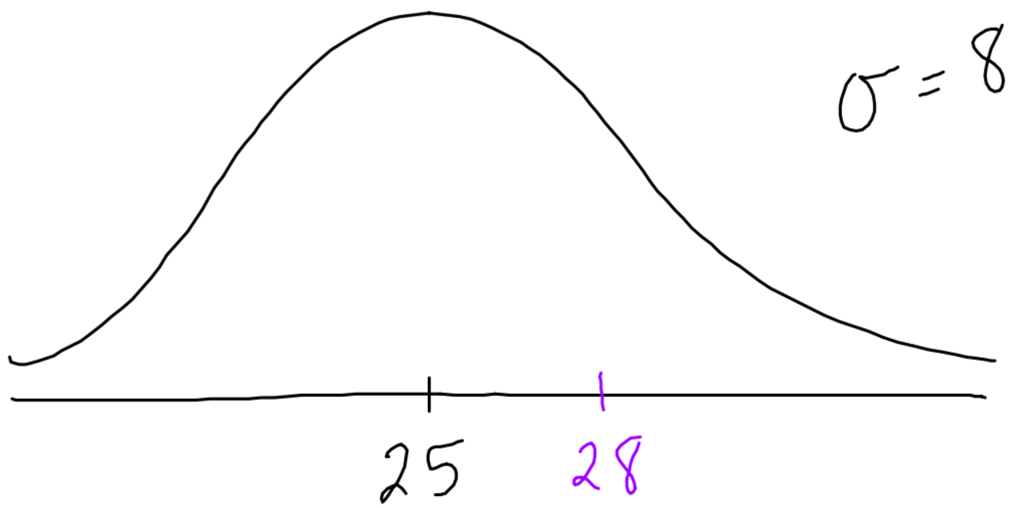
Find $P(X < 20)$



$\sigma = 8$

normalcdf $(0, 20, 25, 8) \approx .2650$

Find $P(X = 28)$

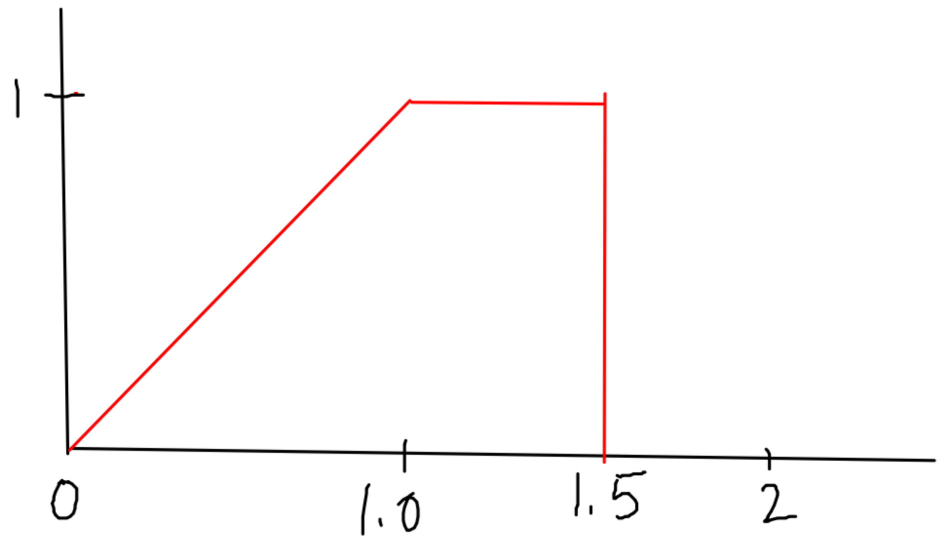


$P(X=28) = 0$ } There is no area above a point!

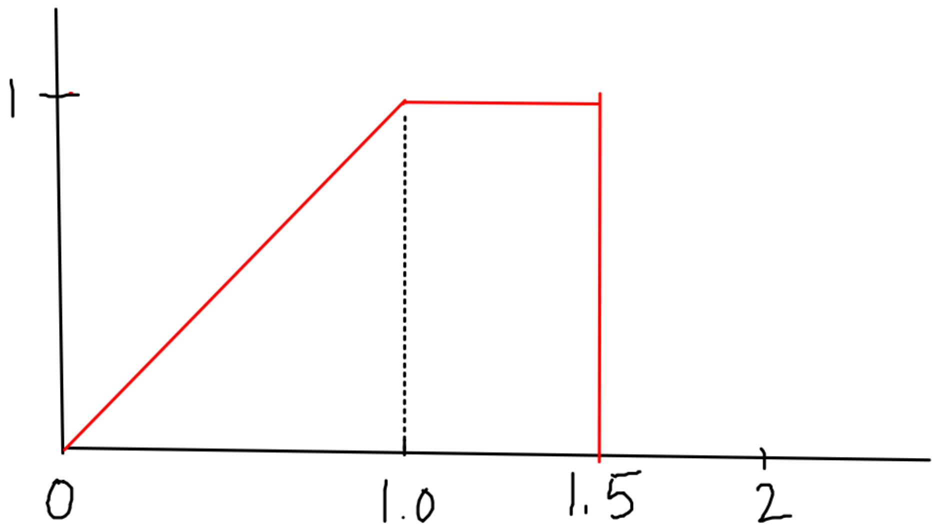
Continuous RV
Example 2

Continuous RVs

A probability density function is comprised of 2 straight line segments. The first begins at (0,0) and goes to (1,1). The second goes from (1,1) to (1.5,1).



1
0
1.0
1.5
2

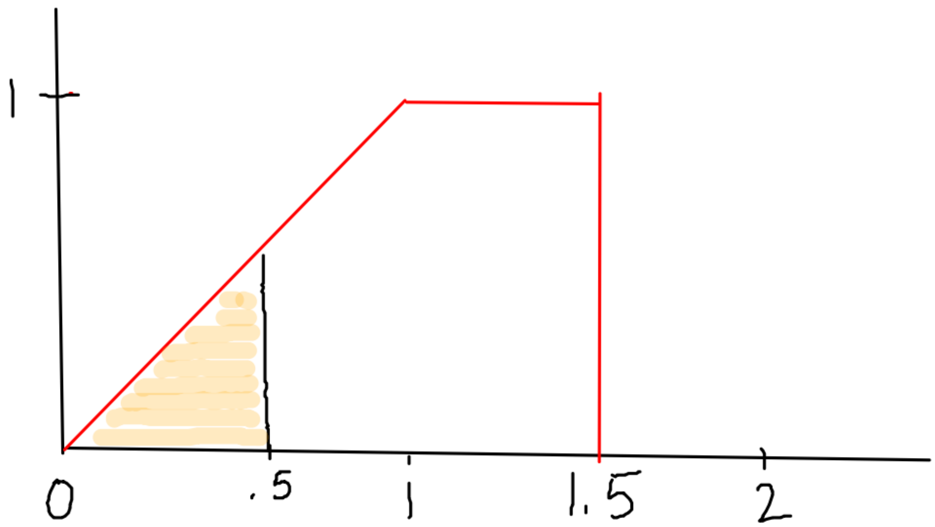


1) Verify this is a legitimate prob distribution

$$\text{Area } \triangle = \frac{1}{2}bh = \frac{1}{2}(1)(1) = .5$$

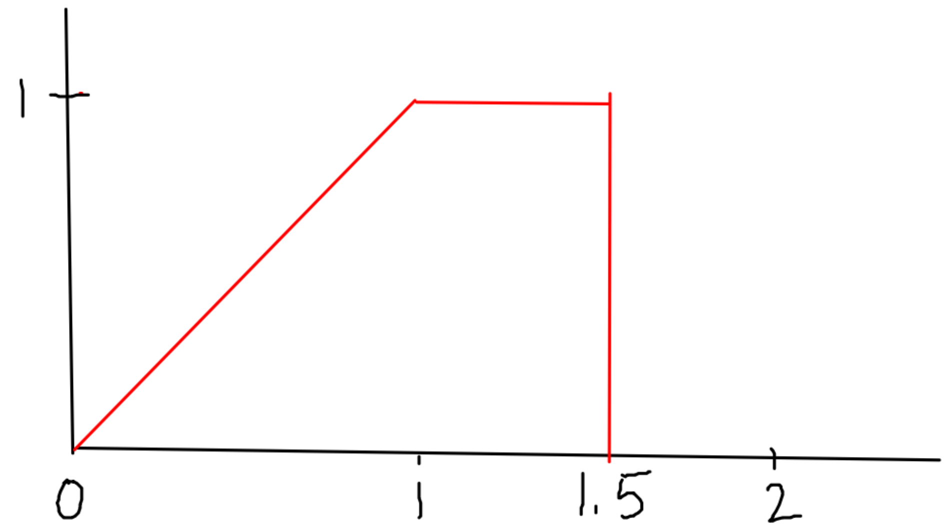
$$\text{Area } \square = bh = (.5)(1) = .5$$

$$1.0 \checkmark$$



2) Find $P(0 < X \leq .5)$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(.5)(.5) = \boxed{.125}$$



3) Find $P(X=1)$

$P(X=1)=0$ } No Area Above A Point!

Discrete RVs

X = the number of ______

Continuous RVs

X = the amount of ______

Ex Game of Chance

If a player rolls 2 dice and gets a sum of 2 or 12, s/he wins \$20. If the person gets a sum of 7, s/he wins \$5. The cost to play is \$3. Find the expected payout.

$X = \text{Payout}$

X	\$20	\$5	\$0
$P(x)$	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{28}{36}$

$$\mu_X = 20\left(\frac{2}{36}\right) + 5\left(\frac{6}{36}\right) + 0\left(\frac{28}{36}\right) = \$1.94$$

Is the game fair?

6.2

Learning Targets

- Find mean/standard deviation of a linearly transformed RV
- Calculate sums/differences for means and standard deviations of RVs

Prob	.03	.16	.30	.23	.17	.11		
X	0	1	2	3	4	5	$\mu_X =$ 2.68	$\sigma_X =$ 1.31
$Y = X+2$	2	3	4	5	6	7	$\mu_Y =$ 4.68	$\sigma_Y =$ 1.31
$Z = 3X$	0	3	6	9	12	15	$\mu_Z =$ 8.04	$\sigma_Z =$ 3.93

Prob	.03	.16	.30	.23	.17	.11		
X	0	1	2	3	4	5	$\mu_X =$ 2.68	$\sigma_X =$ 1.31
Y=X+2	2	3	4	5	6	7	$\mu_Y =$ 4.68	$\sigma_Y =$ 1.31
Z=3X	0	3	6	9	12	15	$\mu_Z =$ 8.04	$\sigma_Z =$ 3.93

×3 +2 NC ×3

Random Variable Rules

i) If $Y = bX + a$, then:

$$\mu_Y = b\mu_X + a$$

$$\sigma_Y = |b|\sigma_X$$

$$\sigma^2_Y = b^2\sigma^2_X$$

Ex Let $y = .9x - .2$

a) If $\mu_x = 3$, find μ_y:

$$\mu_y = .9(3) - .2 = 2.5$$

b) If $\sigma_x = 4$, find σ_y

$$\sigma_y = .9(4) = 3.6$$

Ex Assume $\sigma_x = 20$; Find a and b such that $Y = bX + a$ has a standard deviation of 1.

$$\sigma_y = 1$$

$$b(\sigma_x) = 1$$

$$b(20) = 1$$

$$b = \frac{1}{20}, \quad a = \{\text{real } \#\}$$

2) $\mu_{x \pm y} = \mu_x \pm \mu_y$

x = SAT Math Score ($\mu_x = 625$)

y = SAT Verbal Score ($\mu_y = 590$)

$\mu_{x+y} = \mu_x + \mu_y = 625 + 590 = 1215$

$\mu_{x-y} = \mu_x - \mu_y = 625 - 590 = 35$

3) $\sigma^2_{x \pm y} = \sigma^2_x + \sigma^2_y$ But $\sigma_{x \pm y} \neq \sigma_x + \sigma_y$

Ex If $\sigma_x = 10$ and $\sigma_y = 8$, find σ_{x-y}

$$\sigma_{x-y} = \sqrt{10^2 + 8^2} = \sqrt{164} = 12.8$$

↑ Add Variances

2002 AP Exam

Section II **Part A**

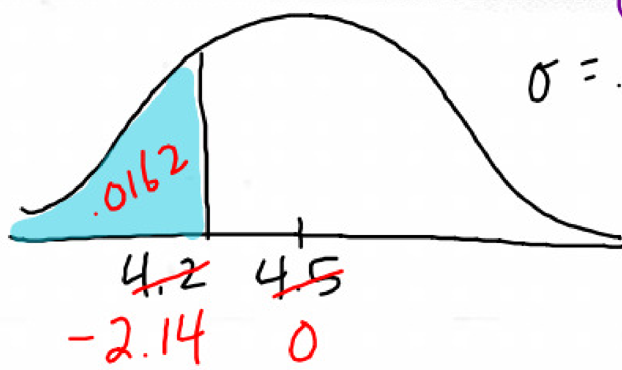
3. There are 4 runners on the New High School team. The team is planning to participate in a race in which each runner runs a mile. The team time is the sum of the individual times for the 4 runners. Assume that the individual times of the 4 runners are all independent of each other. The individual times, in minutes, of the runners in similar races are approximately normally distributed with the following means and standard deviations.

	Mean	Standard Deviation
Runner 1	4.9	0.15
Runner 2	4.7	0.16
Runner 3	4.5	0.14
Runner 4	4.8	0.15

(a) Runner 3 thinks that he can run a mile in less than 4.2 minutes in the next race. Is this likely to happen? Explain.

$\sigma = .14$

Not too likely (<2% chance)



$$\text{normalcdf}(0, 4.2, 4.5, .14) \approx .0160$$

(b) The distribution of possible team times is approximately normal. What are the mean and standard deviation of this distribution?

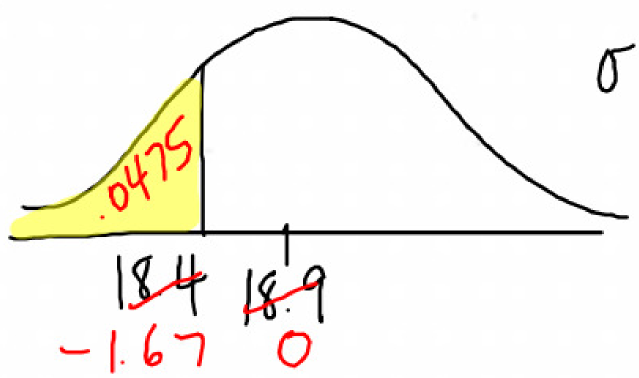
$$\mu_T = 4.9 + 4.7 + 4.5 + 4.8 = 18.9 \text{ min}$$

$$\sigma_T \neq .15 + .16 + .14 + .15\,!$$

$$\sigma_T = \sqrt{(.15)^2 + (.16)^2 + (.14)^2 + (.15)^2} = .3003 \text{ min}$$

(c) Suppose the team's best time to date is 18.4 minutes. What is the probability that the team will beat its own best time in the next race?

$\sigma = .3003$



$$\text{normalcdf}(0, 18.4, 18.9, .3003) \approx .0479$$

Sec 6.3

Learning Targets

- Recognize a Binomial Setting
- Calculate probabilities from a Binomial Setting

Binomial Setting

- 2 outcomes (success/failure)
- P(success) constant
- Fixed number of independent trials (n)

Binomial Random Variable

X = # successes

40 AP Exam Questions $\rightarrow$ Randomly Guess ☹

1) 2 Outcomes (Success / Failure)

2) P(success) = .2 (Constant)

3) Guesses are independent

4) X = # correct guesses (out of 40)

Notation

$$B(n, p) = B(40, .2)$$

n: # of observations

p: probability of success

1) What is the probability that $X = 10$

a) Use binomial formula

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(X=10) = \binom{40}{10} (.2)^{10} (.8)^{30} \approx .1074$$

$40 \rightarrow \boxed{\text{MATH}} \rightarrow \text{PRB} \rightarrow {}_nC_r \rightarrow 10$

b) Use Calculator

$\boxed{\text{DISTR}} \to \text{binompdf}(\underset{n}{40}, \underset{p}{.2}, \underset{k}{10}) \approx .1074$

2) Find $P(X=0)$

a) Formula

$$P(X=0) = \binom{40}{0}(.2)^0(.8)^{40} = .0001329$$

b) Calculator

binompdf$(40, .2, 0) = .0001329$

3) Find $P(X \le 10)$

a) Formula Inefficient

X	0	1	2	...	10
P(X)					

b) Use Calculator

binomcdf(40, .2, 10) ≈ .8392

↑

cumulative for ≤ K

3) Find $P(X \leq 10)$ $P(X < 10)$

a) Formula Inefficient

X	0	1	2	...	10
P(X)					

b) Use Calculator

binomcdf$(40, .2, \cancel{10})$ ≈ .8392
9

cumulative for ≤ K

4) Find $P(X \geq 10)$

a) Formula <u>Really</u> Inefficient 

b) Use calculator/complement rule

$P(X \geq 10) = 1 - \text{binomcdf}(40, .2, 9) \approx .2682$

4) Find $P(X \geq 10)$

a) Formula <u>Really</u> Inefficient 

b) Use calculator / complement rule

$P(X \geq 10) = 1 - \text{binomcdf}(40, .2, 9) \approx .2682$

$P(X > 10) = 1 - \text{binomcdf}(40, .2, 10) \ldots$

Review

For a random variabe X:

$$\mu_X = \sum x_i p_i$$

$$\sigma^2_X = \sum (x_i - \mu_X)^2 p_i$$

If $B(n, p)$ then

$$\mu_x = np = (40)(.2) = 8$$

$$\sigma_x^2 = np(1-p) = (40)(.2)(.8) = 6.4$$

$$\sigma_x = \sqrt{np(1-p)} = \sqrt{6.4} = 2.5$$

If $B(n, p)$ and $np \geq 10$, $n(1-p) \geq 10$

then normal distribution rules can be used...

Ex Attitudes Toward Shopping (P.396)

$B(2500, .6)$

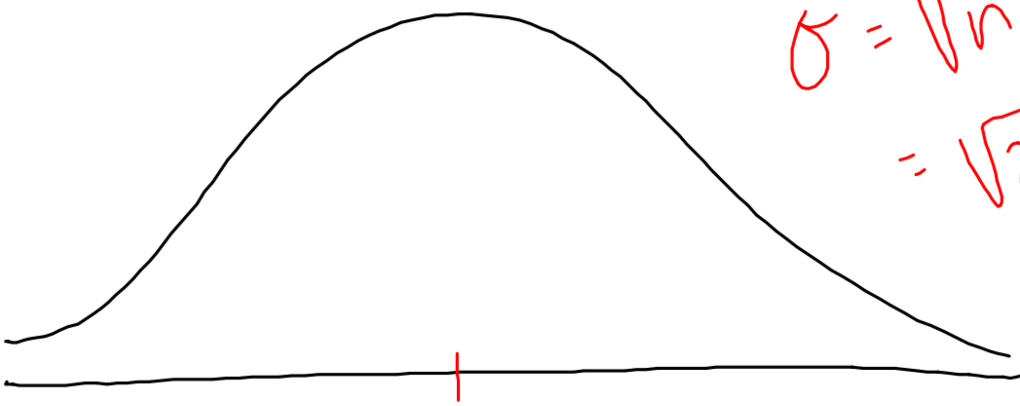
Find $(X \geq 1520)$

Since $np = (2500)(.6) = 1500 > 10$

and $n(1-p) = (2500)(.4) = 1000 > 10$

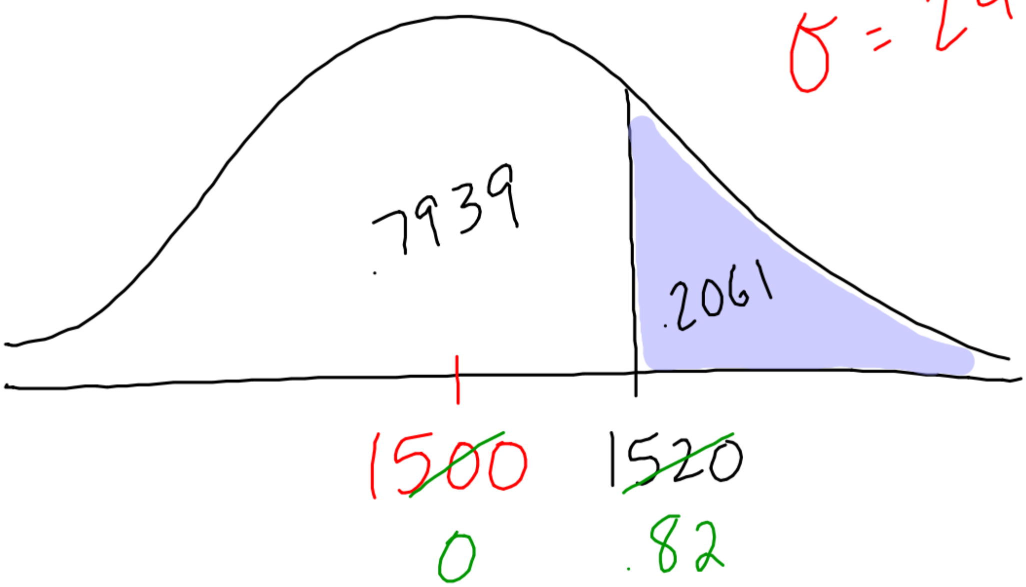
then $P(X > 1520)$ can be found

using a normal distribution . . .



$$\sigma = \sqrt{np(1-p)}$$
$$= \sqrt{2500(.6)(.4)}$$
$$= 24.49$$

$$\mu = np = (2500)(.6) = 1500$$



$\sigma = 24.49$

$\text{normalcdf}(1520, 10000, 1500, 24.49) \approx .2071$

$1 - \text{binomcdf}(2500, .6, 1519) \approx .2131$

Regardless of the method used, the probability that 1520 (or more) people in this sample agree that shopping is frustrating is approximately 21%

Sec 6.3 (cont)

Learning Targets

- Recognize a Geometric Setting
- Calculate probabilities from a Geometric Setting

Geometric Setting

- 2 Outcomes (Success / Failure)
- P(success) is constant
- observations independent

Same As Binomial

☆ Geometric Random Variable

X = # trials required before you get the 1st success

EXAMPLE 8.17 ROLL A DIE

The rule for calculating geometric probabilities can be used to construct a probability distribution table for X = number of rolls of a die until a 3 occurs:

X	1	2	3	4	5	6	7	...
P(X)	0.1667	0.1389	0.1157	0.0965	0.0804	0.0670	0.0558	...

Here's one way to find these probabilities with your calculator:

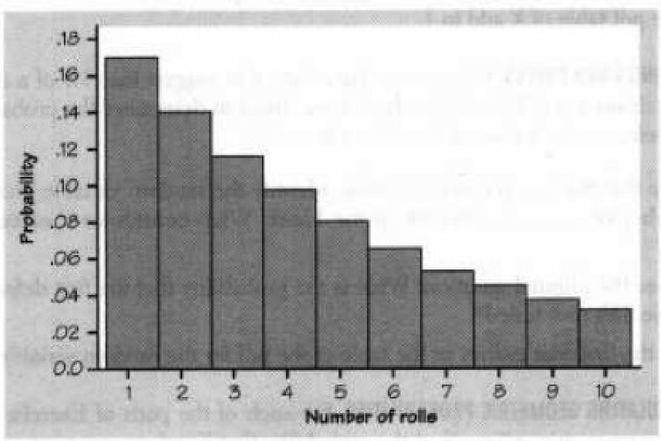
1. Enter the probability of success, 1/6. Press ENTER.
2. Enter *(5/6) and press ENTER.
3. Continue to press ENTER repeatedly.

```
1/6
          .1666666667
Ans*(5/6)
          .1388888889
          .1157407407
          .0964506173
          .0803755144
```

Verify that the entries in the second row are as shown:

X	1	2	3	4
P(X)	1/6	5/36	25/216	125/1296

Figure 8.4 is a graph of the distribution of X. As you might expect, the probability distribution histogram is strongly skewed to the right with a peak at the leftmost value, 1. It is easy to see why this must be so, since the height of each bar after the first is the height of the previous bar times the probability of failure $1 - p$. Since you're multiplying the height of each bar by a number less than 1, each new bar will be shorter than the previous bar, and hence the histogram will be right-skewed. Always.



skewed right always

FIGURE 8.4 Probability histogram for the geometric distribution.

Ex Roll a die...

1) Find P(Get a 3 on 4th Roll)

geometpdf $\left(\frac{1}{6}, 4\right) \approx .0964$

From last chapter:

$P(X=4) = P(No) \cdot P(No) \cdot P(No) \cdot P(Yes)$

$= 5/6 \cdot 5/6 \cdot 5/6 \cdot 1/6$

$= .0964$

2) Find P(Get a 3 on one of 1st 4 rolls)

geometcdf$\left(\frac{1}{6}, 4\right) \approx .5177$

3) Find P(Takes more than 4 rolls to get a 3)

$$P(X>4) = 1 - \text{geometcdf}\left(\frac{1}{6}, 4\right) \approx .4822$$

From last chapter:

$$P(X>4) = P(\text{No}) \cdot P(\text{No}) \cdot P(\text{No}) \cdot P(\text{No})$$

$$= (5/6)(5/6)(5/6)(5/6)$$

$$= .4822$$

Success/Failure

P (success) constant

Independence

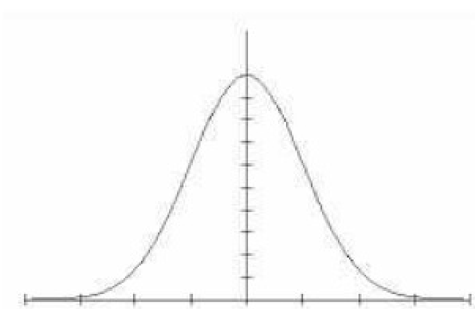
Binomial

Count number of successes from a fixed number of trials

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$

$P(X=k)=$ binom**pdf** (n, p, k)

$P(X\le k)=$ binom**cdf** (n, p, k)



$$[\,np\ge 10 \qquad n(1-p)\ge 10\,]$$

$$\mu = np$$

$$\sigma=\sqrt{np(1-p)}$$

Geometric

Count number of trials needed to obtain first success

$$P(\textit{success on } n^{th} \textit{ trial}) = p(1-p)^{n-1}$$

$P(X=n)=$ geomet**pdf** (p, n)

$$P(X>n)=(1-p)^n$$

$P(X\le n)=$ geomet**cdf** (p, n)

$$\mu=\frac{1}{p}$$