Sec 6.1

Variables

$$\triangle \to x \to \sigma$$

Random Variable (X)

A variable whose value is a probability

Discrete RVs

The probabities that a customer selects 1, 2, 3, 4 or 5 items is :

X	1	2	3	4	5
P(X)	.32	.12	.23	.18	.15

= 1.00 ✓

X = the number of items bought

$$P(X > 3.5) = P(X = 4 \text{ or } 5) = .33$$

$$P(1.0 < X < 3.0) = P(x = 2) = .12$$

$$P(X < 5) = 1 - P(X = 5) = .85$$

Mean of a RV (Weighted Average)

$$\boxed{E(X) = \mu_X = \sum x_i p_i}$$

$$\mu_X = 1(.32) + 2(.12) + 3(.23) + 4(.18) + 5(.15)$$

$$= \boxed{2.72 \text{ items}}$$

Don't Round

Variance of a RV

$$\boxed{Var(x) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i}$$

$$\sigma^2 = (1-2.72)^2(.32) + (2-2.72)^2(.12) + (3-2.72)^2(.23)$$
$$+ (4-2.72)^2(.18) + (5-2.72)^2(.15) = 2.1016$$

$$\sigma = \sqrt{2.1016} = 1.4496$$

Finding Mean/Standard Deviation Using Calculator:

L_1 = X Values

L_2 = Probabilities

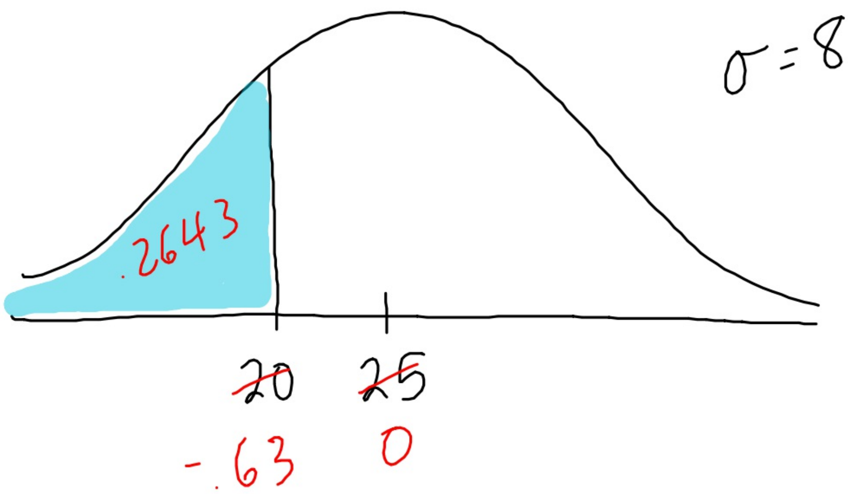
1 Var Stats $\rightarrow L_1, L_2$ (Frequency)

Continuous RVs

The time it takes Mark to do his math homework follows a Normal distribution with mean 25 minutes and standard deviation 8 minutes

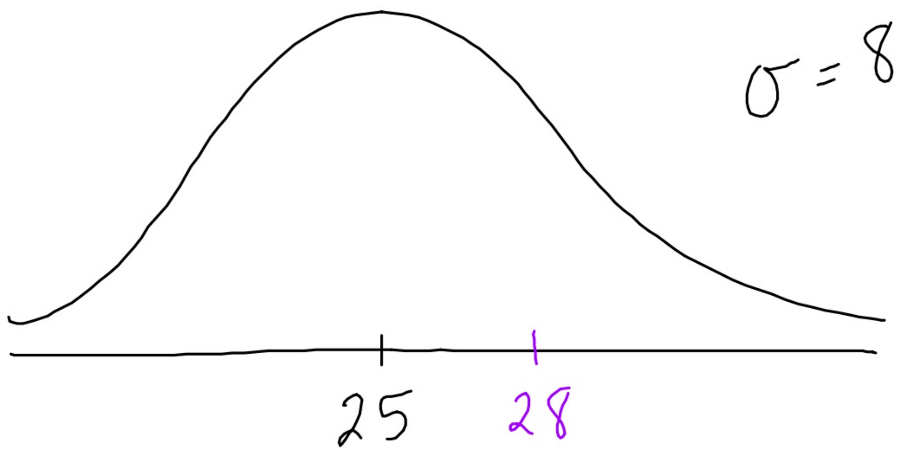
Let X = time to complete math homework

Find $P(x < 20)$

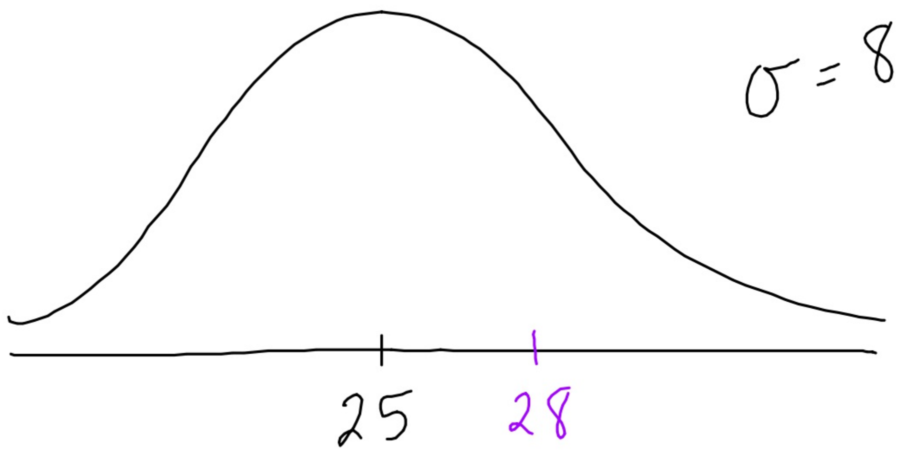


normalcdf$(0, 20, 25, 8) \approx .2650$

Find $P(X = 28)$



Find $P(X = 28)$

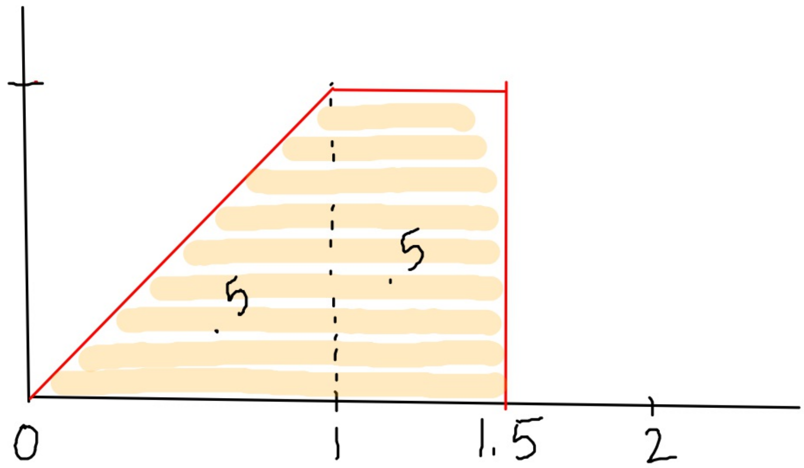


$P(X = 20) = 0$ There is no area above a point!

Continuous RV
Example 2

Continuous RVs

A probability density function is comprised of 2 straight line segments. The first begins at $(0,0)$ and goes to $(1,1)$. The second goes from $(1,1)$ to $(1.5,1)$.



1) Verify this is a legitimate prob distribution

$A = (.5) + (.5) = 1$ ✓

2) $P(0 < X \le .5)$

$= \frac{1}{2}(.5)(.5) = .125$

3) $P(X = 1) = 0!$

No Area
Above A Point

Discrete RVs

X = the number of _____

Continuous RVs

X = the amount of _____

Ex Game of Chance

If a player rolls 2 dice and gets a sum of 2 or 12, s/he wins \$20. If the person gets a sum of 7, s/he wins \$5. The cost to play is \$3. Find the expected payout.

$X = \text{Payout}$

X	\$20	\$5	\$0
$P(x)$	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{28}{36}$

$$\mu_X = 20\left(\frac{2}{36}\right) + 5\left(\frac{6}{36}\right) + 0\left(\frac{28}{36}\right) = \$1.94$$

Is the game fair?

6.2

	Data Set	$\bar{X}$		S	
	{2, 5, 8}	5		3	
+4	{6, 9, 12}	9	+4	3	NC
x2	{4, 10, 16}	10	x2	6	x2

Rules For Random Variables

1) $Y = bX + a$

$$\mu_Y = b\mu_X + a$$

$$\sigma_Y = b\sigma_X$$

$$\sigma^2_Y = b^2\sigma^2_X$$

Ex Let $Y = .9X - .2$

If $\mu_x = 3$, find μ_y:

$\mu_y = .9(3) - .2 = 2.5$

If $\sigma_x = 4$, find σ_y

$\sigma_y = .9(4) = 3.6$

Ex Assume $\sigma_x = 20$; Find a and b such that $Y = bX + a$ has a standard deviation of 1.

$$\sigma_y = 1$$

$$b(\sigma_x) = 1$$

$$20b = 1$$

$$b = \frac{1}{20}$$

$$a = \{\text{real}\}$$

2) $M_{x \pm y} = M_x \pm M_y$

X = SAT Math Score ($M_x = 625$)

Y = SAT Verbal Score ($M_y = 590$)

$M_{x+y} = M_x + M_y = 625 + 590 = 1215$

$M_{x-y} = M_x - M_y = 625 - 590 = 35$

3) $\sigma^2_{x \pm y} = \sigma^2_x + \sigma^2_y$ But $\sigma_{x+y} \neq \sigma_x + \sigma_y$

Ex If $\sigma_x = 10$ and $\sigma_y = 8$, find σ_{x+y}

$$\sigma^2_{x+y} = \sigma^2_x + \sigma^2_y$$
$$= 100 + 64$$
$$= 164$$

$$\sigma_{x+y} = \sqrt{164} \approx 12.8$$

Ex 2002 AP Exam

Relay Team	Mean (Mile)	Stand Dev
1	4.9 min	.15 min
2	4.7	.16
3	4.5	.14
4	4.8	.15

1) What is the team's total mean time?

$$\mu_T = 4.9 + 4.7 + 4.5 + 4.8 = 18.9 \text{ min}$$

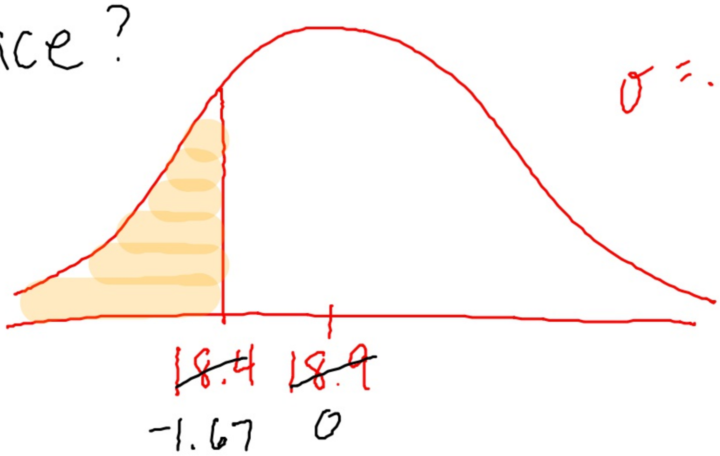
2) What is the stand dev of the team?

$$\sigma^2_T = (.15)^2 + (.16)^2 + (.14)^2 + (.15)^2 = .0902$$

$$\sigma_T = \sqrt{.0902} \approx .3003 \text{ min}$$

3) Suppose the team's best time to date is 18.4 min. If the distributim is aprox normal, what is the probability that the team will beat its own best time in the next race?

$\sigma \approx .3003$



normalcdf(0, 18.4, 18.9, .3003) ≈ .0479

Sec 6.3

Binomial Setting

- 2 outcomes (success/failure)
- P(success) constant
- Fixed number of independent trials (n)

Binomial Random Variable

X = # successes

Randomly guess answers → 40 SAT questions

1) 2 Outcomes (Success / Failure)

2) P(success) = .2 (Constant)

3) Guesses are independent

4) X = # correct guesses (out of 40)

Notation

$$B(n, p) = B(40, .2)$$

\# of observations

probability of success

1) What is the probability that $X = 10$

a) Use binomial formula

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(X=10) = \binom{40}{10} (.2)^{10} (.8)^{30} \approx .1074$$

$40 \rightarrow \boxed{\text{MATH}} \rightarrow \text{PRB} \rightarrow 10$

b) Calculator Function

$\boxed{\text{DISTR}} \rightarrow \text{binompdf}(\underset{n}{40}, \underset{p}{.2}, \underset{K}{10}) = .1075$

2) Find $P(X=0)$

a) $P(X=0) = \binom{40}{0}(.2)^0(.8)^{40} = .0001$

b) $\text{binompdf}(40, .2, 0) = .0001$

3) Find $P(X \leq 10)$

a) Formula Inefficient

X	0	1	2	...	10
$P(X)$					

b) Use Calculator

binomcdf(40, .2, 10) $\approx$ .8392

↑

cumulative
for $\leq K$

3) Find $P(x \leq 10)$ $\quad P(X < 10)$

a) Formula Inefficient

X	0	1	2	...	10
P(x)					

b) Use Calculator

$$\text{binomcdf}(40, .2, \underset{9}{\cancel{10}}) \approx .8392$$

cumulative
for $\leq K$

4) Find $P(X \geq 10)$

a) Formula Really Inefficient

b) Use calculator / complement rule

$P(X \geq 10) = 1 - \text{binomcdf}(40, .2, 9) \approx .2682$

Review

For a random variabe X:

$$\mu_X = \sum x_i p_i$$

$$\sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$$

If $B(n,p)$ then

$$\mu_x = np = (40)(.2) = 8$$

$$\sigma^2_x = np(1-p) = (40)(.2)(.8) = 6.4$$

$$\sigma_x = \sqrt{n(p)(1-p)} = \sqrt{6.4} = 2.5298$$

If $B(n, p)$ and $np \geq 10$, $n(1-p) \geq 10$

then normal distribution rules can be used...

Ex Attitudes Toward Shopping (P. 396)

$B(2500, .6)$

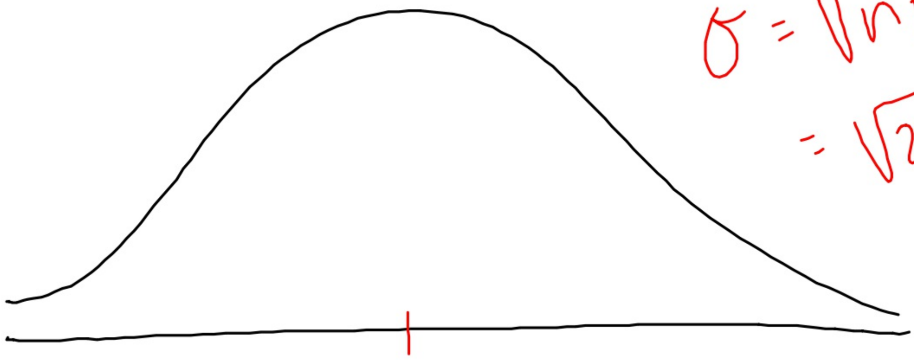
Find $(X \geq 1520)$

Since $np = (2500)(.6) = 1500 > 10$

and $n(1-p) = (2500)(.4) = 1000 > 10$

then $P(X > 1520)$ can be found

using a normal distribution . . .

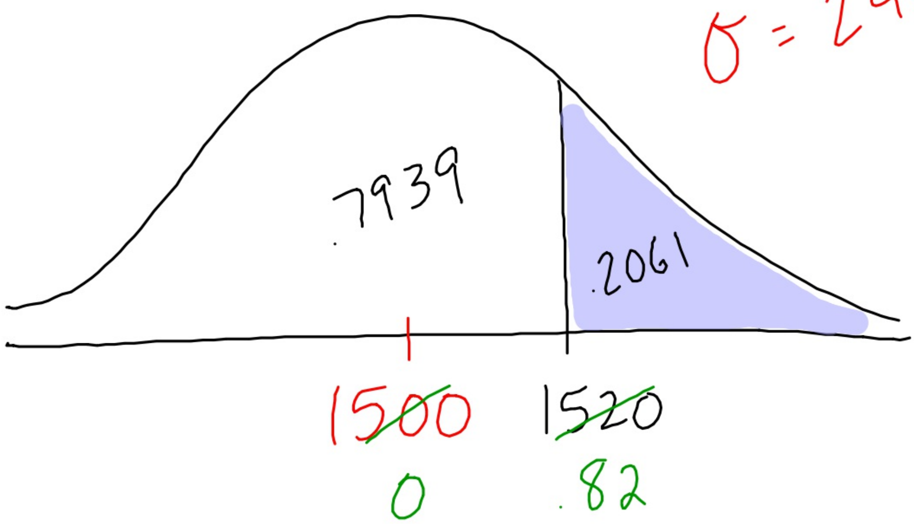


$\sigma = \sqrt{np(1-p)}$

$= \sqrt{2500(.6)(.4)}$

$= 24.49$

$\mu = np = (2500)(.6) = 1500$



$\sigma = 24.49$

normalcdf(1520, 10000, 1500, 24.49) ≈ .2071

1 - binomcdf(2500, .6, 1519) ≈ .2131

Regardless of the method used, the probability that 1520 (or more) people in this sample agree that shopping is frustrating is approximately 21%

Sec 6.3 (cont)

Geometric Setting

- 2 Outcomes (Success / Failure)
- P(success) is constant
- observations independent

☆ Geometric Random Variable

X = # trials required before you get the 1st success

Geometric Formulas

$$P\begin{pmatrix}\text{1st Success On} \\ \text{nth Trial}\end{pmatrix} = P(X=n) = p\,q^{n-1}$$

$(1-p)$

As the # trials increase

the probability for the first success decreases...

EXAMPLE 8.17 ROLL A DIE

The rule for calculating geometric probabilities can be used to construct a probability distribution table for X = number of rolls of a die until a 3 occurs:

X	1	2	3	4	5	6	7	...
P(X)	0.1667	0.1389	0.1157	0.0965	0.0804	0.0670	0.0558	...

Here's one way to find these probabilities with your calculator:

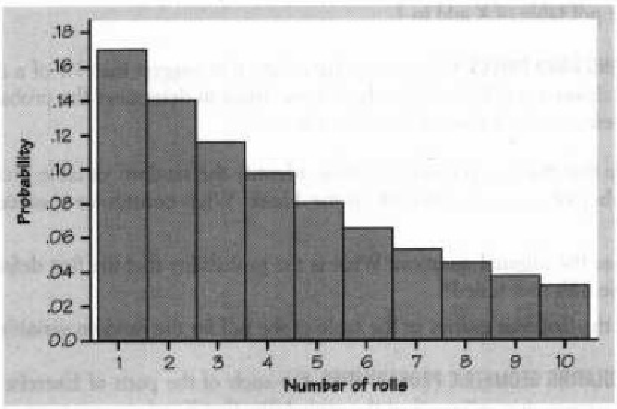
1. Enter the probability of success, 1/6. Press ENTER.
2. Enter *(5/6) and press ENTER.
3. Continue to press ENTER repeatedly.

```
1/6
          .1666666667
Ans*(5/6)
          .1388888889
          .1157407407
          .0964506173
          .0803755144
```

Verify that the entries in the second row are as shown:

X	1	2	3	4
P(X)	1/6	5/36	25/216	125/1296

Figure 8.4 is a graph of the distribution of X. As you might expect, the probability distribution histogram is strongly skewed to the right with a peak at the leftmost value, 1. It is easy to see why this must be so, since the height of each bar after the first is the height of the previous bar times the probability of failure $1 - p$. Since you're multiplying the height of each bar by a number less than 1, each new bar will be shorter than the previous bar, and hence the histogram will be right-skewed. Always.



skewed right always

FIGURE 8.4 Probability histogram for the geometric distribution.

$$P\begin{pmatrix}\text{More than } n \text{ trials} \\ \text{to get 1st success}\end{pmatrix} = P(X > n) = \underset{1-p}{q^n}$$

$$\mu = \frac{1}{p}$$

Ex Roll a die...

1) Find P (Get a 3 on 4th Roll)

a) Using Formula

$P(X=n) = pq^{n-1} = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{4-1} \approx .0964$

b) Using Calculator

geometpdf $\left(\frac{1}{6}, 4\right) \approx .0964$

2) Find P (Get a 3 on one of 1st 4 rolls)

a) Formula Inefficient ⍨

$P(X \leq 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

b) Use Calculator

geometcdf $\left(\frac{1}{6}, 4\right) \approx .5177$

3) Find P(Takes more than 4 rolls to get a 3)

a) Using Formula

$$P(X>n) = q^n = \left(\frac{5}{6}\right)^4 \approx .4822$$

b) Using Calculator

$$P(X>4) = 1 - \text{geometcdf}\left(\frac{1}{6}, 4\right)$$

Success/Failure

P (success) constant

Independence

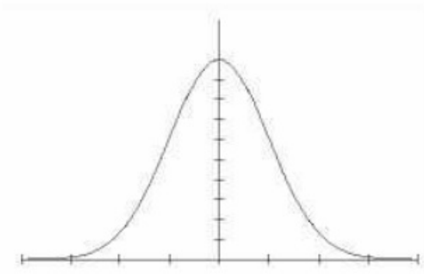
Binomial

Count number of successes from a fixed number of trials

$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$

$P(X=k)=$ binom**pdf** (n, p, k)

$P(X\le k)=$ binom**cdf** (n, p, k)



$[\,np\ge 10 \quad n(1-p)\ge 10\,]$

$\mu = np$

$\sigma=\sqrt{np(1-p)}$

Geometric

Count number of trials needed to obtain first success

$P(\textit{success on } n^{th} \textit{ trial}) = p(1-p)^{n-1}$

$P(X=n)=$ geomet**pdf** (p, n)

$P(X>n)=(1-p)^n$

$P(X\le n)=$ geomet**cdf** (p, n)

$\mu=\dfrac{1}{p}$

AP Formulas