Sec 7.1
Probability Distributions

![Diagram showing a graph with the x-axis labeled "Values" and the y-axis labeled "Probability".]
Types of Probability Distributions

- Normal Distribution (Ch 2)
- Prob Dist of Random Variables (Ch 7)
- Binomial Distributions > Ch 8
- Geometric Distributions
Random Variable \( (X) \)

A variable whose value is a probability of a random phenomena

- Discrete
- Continuous

Probability Distribution

\[ X = \text{the number of } \_ \_ \]

\[ X = \text{the amount of } \_ \_ \]
Ex  Discrete Random Variable

The probabilities that a BP customer selects 1, 2, 3, 4 or 5 items are:

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>.32</td>
<td>.12</td>
<td>.23</td>
<td>.18</td>
<td>.15</td>
</tr>
</tbody>
</table>

P(X > 3.5) = P(X = 4 or 5) = .33
P(1.0 ≤ X ≤ 3.0) = P(X = 2) = .12
P(X ≤ 5) = P(X ≠ 5) = 1 - .15 = .85
Ex Continuous Random Variable

A probability density function is comprised of 2 line segments. The first segment begins at (0,0) and goes to (1,1). The second segment goes from (1,1) to (1.5,1).
1) Verify legitimate density curve (A = 1)

\[
A = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1 \checkmark
\]
2) \( P(0 < x \leq .5) \)

\[ A = \frac{1}{2}bh = \frac{1}{2}(.5)(.5) = .125 \]
3) \( P(1 \leq X \leq 1.25) \)

\[
A = (.25)(1) = .25
\]

4) \( P(X = 1) = 0 \) \( \Rightarrow \) No Area Above A Point!
Continuous Random Variable

Assume the amount of ice cream in an open 5-gallon container is uniformly distributed. What is the probability that the sales person will have to open a new 5-gallon container when you order a quart of ice cream?
\[ P(X < 1) = \frac{1}{20} = 0.05 \]
Notes

- Ignore $\leq \text{ vs } \leq (\geq \text{ vs } \geq)$ for continuous random variables
- $x + x \neq 2x$
normalcdf (.16, 100, .15, .0092) ≈ .1385
7.20b) \[ \text{normalcdf} \ (0.14, 0.16, 0.15, 0.0092) = 0.7242 \]

\[ 0.8621 - 0.1379 = 0.7242 \]
NOTES QUIZ
(Section 7.1)

Name _______________________

CHOOSE ONE OF THE FOLLOWING; SHOW ALL WORK:

1. During rush hour, subway trains run every 10 minutes. If you arrive on the platform at a random time, what is the probability that you will have to wait more than 3 minutes for the next train?

   \[
   \frac{1}{10} \quad 0 \quad 3 \quad 10
   \]

2. Let the random variable \( X \) represent the profit made on a randomly selected day by a certain store. Assuming \( X \) is normal with a mean of $360 and standard deviation of $50, find \( P(X > 400) \):

   \[
   \sigma = 50
   \]

   ![Diagram showing normal distribution with values 360, 400, and 0.80]
Sec 7.2
Mean of Random Variable ($\mu X$)

Ex Size of Households (Ex 7.10)

<table>
<thead>
<tr>
<th>#Inhabitants</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.25</td>
<td>.32</td>
<td>.17</td>
<td>.15</td>
<td>.07</td>
<td>.03</td>
<td>.01</td>
</tr>
</tbody>
</table>

$\mu X = 1(.25) + 2(.32) + 3(.17) + \ldots + 7(.01) = 2.6$

$E(X) = \mu X = \sum X_i p_i$

Expected Value
Ex Game of Chance
If a player rolls 2 dice and gets a sum of 2 or 12, s/he wins $20. If s/he gets a sum of 7, the player wins $5. The cost to play is $3. Find the expected payout for the game.
\[ X = \text{the payout} \]

<table>
<thead>
<tr>
<th>X</th>
<th>$0</th>
<th>$5</th>
<th>$20</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>\frac{28}{36}</td>
<td>\frac{6}{36}</td>
<td>\frac{2}{36}</td>
</tr>
</tbody>
</table>

\[ M_X = 0 \left( \frac{28}{36} \right) + 5 \left( \frac{6}{36} \right) + 20 \left( \frac{2}{36} \right) = 1.94 \]

Is the game fair?
Variance Review (Ch 1)

\[ s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n-1} \]

Standard Deviation

\[ s = \sqrt{\text{Variance}} \]
Variance of Random Variables

\[ \sigma^2_X = (X_1 - M_X)^2 p_1 + (X_2 - M_X)^2 p_2 + \ldots + (X_n - M_X)^2 p_n \]

\[ \text{Var}(X) = \sigma^2_X = \sum (X_i - M_X)^2 p_i \]

\[ \sigma^2_X = (1 - 2.6)^2 (.25) + (2 - 2.6)^2 (.32) + (3 - 2.6)^2 (.17) \]
\[ + (4 - 2.6)^2 (.15) + \ldots + (7 - 2.6)^2 (.01) = 2.02 \]

\[ \sigma_X = \sqrt{2.02} = 1.42 \]
### Ch 1 Review

#### Data Set

<table>
<thead>
<tr>
<th>Set</th>
<th>Operation</th>
<th>New Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>{2, 5, 8}</td>
<td>x _____</td>
<td>_____</td>
</tr>
<tr>
<td>+4 {6, 9, 12}</td>
<td></td>
<td>_____</td>
</tr>
<tr>
<td>x2 {4, 10, 16}</td>
<td></td>
<td>_____</td>
</tr>
<tr>
<td>+4 {8, 14, 20}</td>
<td></td>
<td>_____</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>_____</th>
<th>_____</th>
<th>_____</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>9 + 4</td>
<td>3</td>
<td>NC</td>
</tr>
<tr>
<td>10 x 2</td>
<td>6</td>
<td>x _____</td>
</tr>
<tr>
<td>14 x 2 + 4</td>
<td>6</td>
<td>x _____</td>
</tr>
</tbody>
</table>
Rules For Random Variables

1) \( y = b x + a \)

\[ M_y = b M_x + a \]
\[ \Theta_y = b \Theta_x \]
\[ \Theta^2_y = b^2 \Theta^2_x \]
Ex  Let $M_x = 3$  $O_x = 1.4$ 
and  $Y = .9X - .2$

$My = .9(3) - .2 = 2.5$

$Oy = .9(1.4) = 1.26$
Ex Assume $\sigma_x = 20$

Find $a$ and $b$ such that $y = bX + a$ has a standard deviation of 1.

$\sigma_y = 1$

$b \sigma_x = 1$

$20b = 1$

$b = \frac{1}{20} \rightarrow a = \{\text{reals}\}$
2) \( M_{x+y} = M_{x} + M_{y} \)

\[
X = \text{SAT Math Score} \ (M_x = 625) \\
Y = \text{SAT Verbal Score} \ (M_y = 590) \\
\downarrow \\
M_{x+y} = 625 + 590 = 1215
\]
3) \( O^2_{x+y} = O^2_x + O^2_y \)

**BUT** \( O_{x+y} \neq O_x + O_y \)

**Ex** \( O_x = 10, O_y = 8 \), find \( O_{x+y} \)

\[ O^2_x + O^2_y = 100 + 64 = 164 \]

\[ O_{x+y} = \sqrt{164} = 12.81 \]
### Example

<table>
<thead>
<tr>
<th>HS Relay Team</th>
<th>Mean (Mile)</th>
<th>Stand Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runner 1</td>
<td>4.9 min</td>
<td>.15</td>
</tr>
<tr>
<td>2</td>
<td>4.7</td>
<td>.16</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>.14</td>
</tr>
<tr>
<td>4</td>
<td>4.8</td>
<td>.15</td>
</tr>
</tbody>
</table>

1) What is the team's mean time?

\[ M_T = 4.9 + 4.7 + 4.5 + 4.8 = 18.9 \text{ min} \]

2) What is the team's standard deviation?

\[ \sigma^2_T = (.15)^2 + (.16)^2 + (.14)^2 + (.15)^2 = .0902 \]

\[ \sigma_T = \sqrt{.0902} = .3003 \]