Sec 8.1
Randomly guess answers → 40 MC SAT questions

1) 2 Outcomes (Success / Failure)
2) $P(\text{Success}) = 0.2$ (Constant)
3) Guesses are independent
4) $X = \# \text{ correct guesses out of 40}$

$X = \# \text{ successes from fixed number observations (n)}$
Notation

\[ B(n, p) \rightarrow B(40, 0.2) \]

# observations \[\rightarrow P(\text{Success})\]
1) Find probability that you get exactly 10 questions correct $P(X=10)$

a) Use binomial formula

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{k! (n-k)!}$$

$$P(X=10) = \binom{40}{10} (.2)^{10} (.8)^{30} = .1075$$
b) Use Calculator

\[
\text{DISTR} \rightarrow \text{binompdf} \left( 40, .2, 10 \right) = .1075
\]

2) Find probability that you get 0 correct.
   a) \( P(X=0) = \binom{40}{0} \cdot (.2)^0 \cdot (.8)^{40} = .00013 \)
   b) \( \text{binompdf} \left( 40, .2, 0 \right) = .00013 \)
3) Find probability that you get 10 or less correct \( P(X \leq 10) \)

a) Formula Inefficient

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>\ldots</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X) )</td>
<td></td>
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</tbody>
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b) Calculator

\[
\boxed{\text{DISTR} \rightarrow \text{binomcdf} (40, .2, 10) = .8392}
\]
4) Find probability of getting more than 10 correct $P(X > 10)$
   a) Use Formula ... Probably Not
   b) Use Calculator / Complement Rule

$P(X > 10) = 1 - P(X \leq 10) = 1 - 0.8392 = 0.1608$
If $B(n, p)$ then...

$M_x = np = (40)(.2) = 8$

$\sigma^2_x = np(1-p) = (40)(.2)(.8) = 6.4$

$\sigma_x = \sqrt{\sigma^2_x} = \sqrt{6.4} = 2.5298$
If $B(n,p)$ and...

$$np \geq 10 \quad n(1-p) \geq 10$$

then normal distribution rules can be used
Ex: $B(2500, .6)$, find $P(X \geq 1520)$

$np = (2500)(.6) = 1500 \geq 10$ ✓
$n(1-p) = (2500)(.4) = 1000 \geq 10$ ✓

$Z = \sqrt{\frac{(2500)(.6)(.4)}{1500}} = 24.49$

1500 1520

a) $\text{normalcdf}(1520, 10000, 1500, 24.49) \approx .2071$

b) $1 - \text{binomcdf}(2500, .6, 1519) \approx .2131$
Sec 8.2
Geometric Setting

1) 2 outcomes - success/failure
2) \( P(\text{success}) = \text{constant} \)
3) Observations independent
4) \( X = \# \text{ trials required to get 1st success} \)
Binomial
- Number of trials fixed (n)
- Number of successes varies

Geometric
- Number of trials varies
- Number of successes fixed (at one)
Geometric Probability Formulas

\[ P(X = n) = p q^{n-1} \]

1st Success on \( n \)th trial

\[ P(X > n) = q^n \]

More than \( n \) trials before 1st Success
Mean / Standard Deviation Formulas

\[ M_x = \frac{1}{P} \]

\[ O_x = \sqrt{\frac{1-P}{P^2}} = \frac{\sqrt{1-P}}{P} \]
Ex  Roll a standard die

1) Determine the probability of getting a "3" on the second roll

a) Use Formula

\[ P(X=2) = \left( \frac{1}{6} \right)^2 \cdot \frac{5}{6} = 0.1389 \]

b) Use calculator

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DISTR \rightarrow \text{geompdf} \left( \frac{1}{6}, 2 \right) = 0.1389
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2) Determine probability of getting a "3" on 1st, 2nd, 3rd or 4th roll.

a) Use formula?
   \[ P(X \leq 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4) \]

b) Use calculator
   \[
   \text{DISTR} \rightarrow \text{geometric} \ (1/6, \ 4) = 0.5177
   \]
3) Determine probability that it takes more than 20 rolls before getting a "3"

\[ P(X > 20) = \left( \frac{5}{6} \right)^{20} = 0.0261 \]
4) How many times would you expect to roll a die before getting your first "3"?

\[ M = \frac{1}{P} = \frac{1}{\frac{1}{6}} = 6 \]