Sec 11-2

Sequence

Pattern of numbers, letters, or objects (IQ Tests)

Ex $1, 4, 9, 16, 25, 36 \ldots$

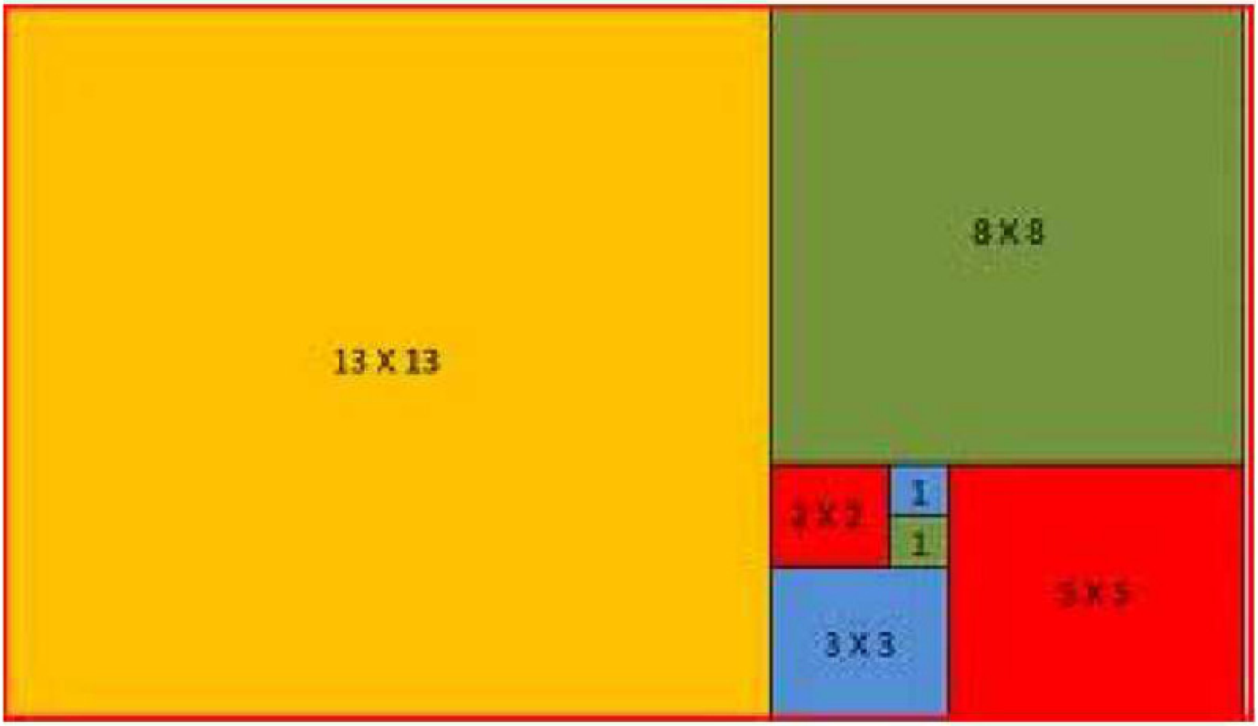
1st Term a_1

6th Term a_6

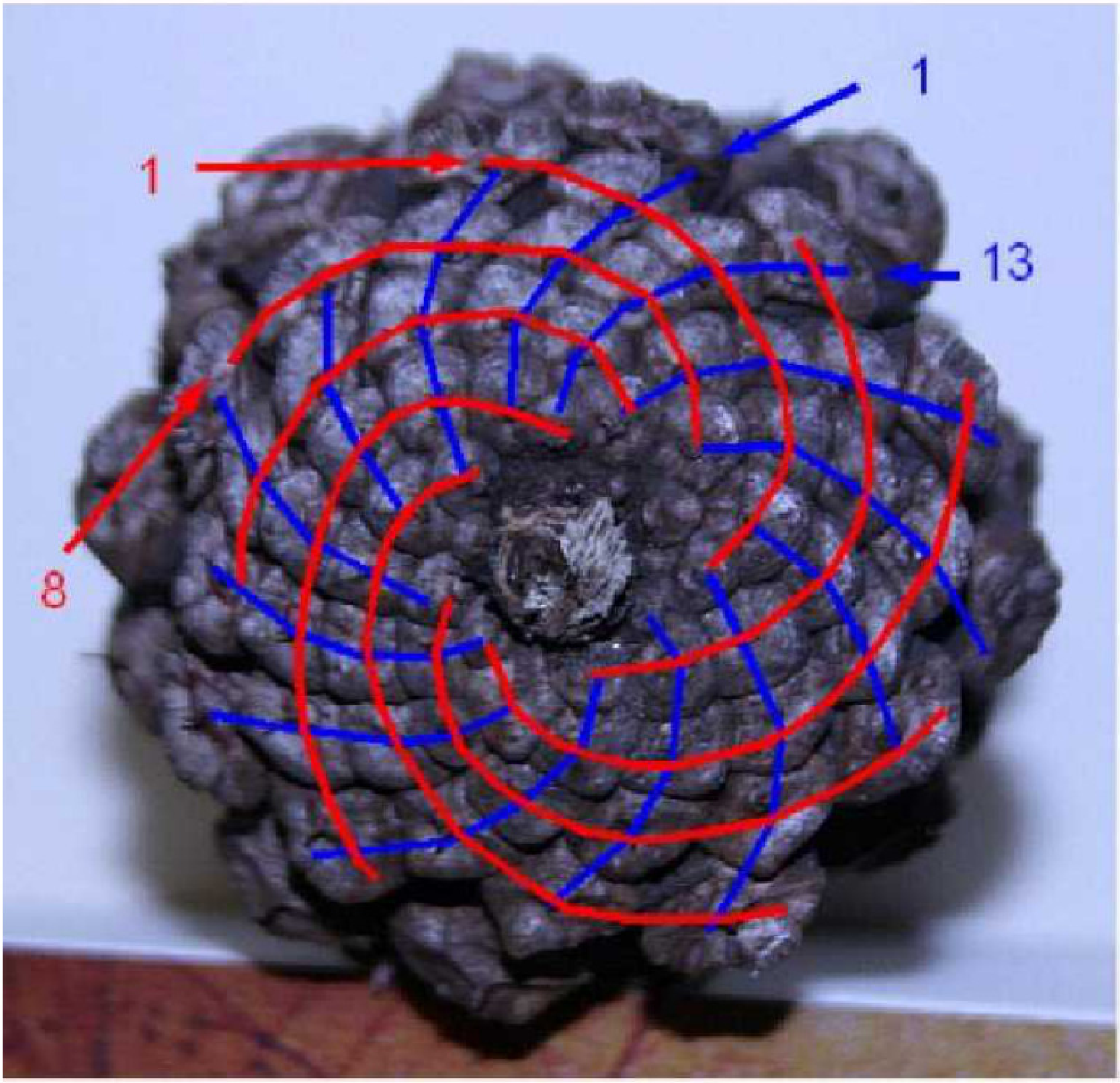
Fibonacci Sequence

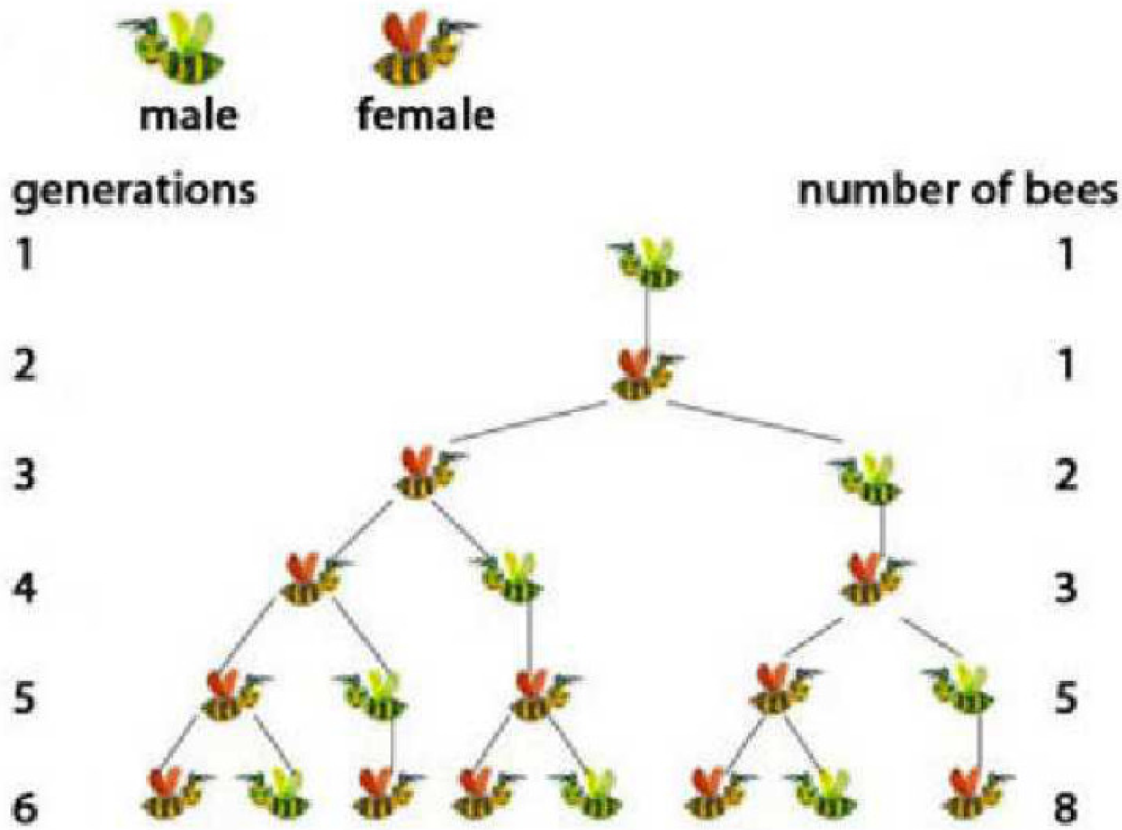
1, 1, 2, 3, 5, 8 ...

Like π and Φ this pattern can be found in art, architecture and nature!



This pattern of "**Fibonacci square**" continues on in the painting of Mona Lisa **...**


1
1
13
8



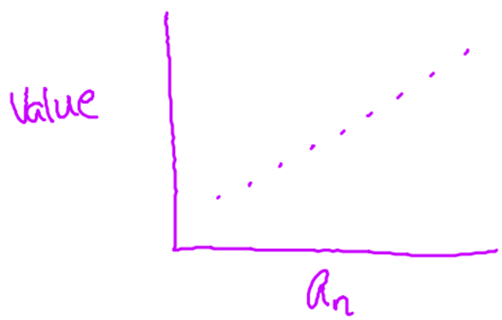
The queen bee has control over whether she lays male or female eggs, and she lays male eggs in slightly larger cells. If she uses stored sperm to fertilize the egg first, the larva that hatches is female. If she leaves the egg unfertilized, the larva that hatches is male. This means that female bees inherit genes from their mothers and their fathers while male bees inherit only genes from their mothers.

Arithmetic Sequence

Each term is obtained by adding a constant (common difference) to each preceding term

Exs 4, 7, 10, 13, 16, 19 ... $d = 3$

20, 18, 16, 14, 12 ... $d = -2$



Formula

$$a_n = a_1 + (n-1)d$$

Ex $a_1 = 4,\ a_2 = 7,\ a_{25} = \_\_\_\_$

$$a_n = a_1 + (n-1)d$$

$$a_{25} = 4 + (25-1)(3)$$

$$a_{25} = 76$$

Ex $a_n = 68,\ a_1 = 5,\ d = 3,\ n = \_\_\_\_$

$$a_n = a_1 + (n-1)d$$

$$68 = 5 + (n-1)(3)$$

$$63 = 3n - 3$$

$$22 = n$$

$$22 = n$$

Arithmetic Means

Missing term(s) in an arithmetic sequence

Ex ... 5, ___, 15, ...

$$\frac{5+15}{2} = 10$$

Ex ... 55, __, __, __, __, 85, ...

a_1 a_6

$$a_n = a_1 + (n-1)d$$

$$85 = 55 + (6-1)d$$

$$30 = 5d$$

$$6 = d$$

Sec 11-3

Geometric Sequence

Each term is obtained by multiplying a constant (common ratio) to each preceding term

Ex $2, 4, 8, 16, 32, 64 \ldots \quad r = 2$ 

$100, 50, 25, \frac{25}{2}, \frac{25}{4} \cdots \quad r = \frac{1}{2}$

Formula

$$a_n = a_1 \cdot r^{n-1}$$

Ex $a_1 = 4$, $r = 2$, $a_{10} =$ ______

$$a_n = a_1 \cdot r^{n-1}$$

$$a_{10} = (4)(2)^{10-1}$$

$$a_{10} = (4)(2)^9$$

$$a_{10} = 2048$$

Ex $a_1 = 17,\ r = 2,\ a_n = 34816,\ n = \_\_\_\_$

$$a_n = a_1 \cdot r^{n-1}$$

$$34816 = (17)(2)^{n-1}$$

$$2048 = (2)^{n-1}$$

$$(\underline{2})^{11} = (\underline{2})^{n-1}$$

$$11 = n-1$$

$$12 = n$$

$$a_n = a_1 \cdot r^{n-1}$$

$$34816 = (17)(2)^{n-1}$$

$$\log 2048 = \log 2^{n-1}$$

$$\frac{\log 2048}{\log 2} = \frac{n-1\,(\cancel{\log 2})}{\cancel{\log 2}}$$

$$11 = n-1$$

$$12 = n$$

Geometric Means

Missing term(s) from a geometric sequence

<u>Ex</u> ... 2, ____, 32 ...

$\sqrt{2 \cdot 32} = \sqrt{64} = \pm 8$

Ex

$\ldots\ 7,\ \_\_\_,\ \_\_\_,\ 56\ \ldots$

a_1 a_4

$$a_n = a_1 \cdot r^{n-1}$$

$$56 = (7)(r)^{4-1}$$

$$8 = r^3$$

$$2 = r$$

Sec 11-4

Series

An indicated sum of terms of a sequence

Ex $1, 1, 2, 3, 5, 8, 13, 21 \ldots$

Partial Sums

$S_1 = 1$

$S_4 = 1 + 1 + 2 + 3 = 7$

$S_6 = 1 + 1 + 2 + 3 + 5 + 8 = 20$

S_n = sum of 1st n terms

Summation Notation

$$\sum_{n=1}^{5}(n^2-4) = (-3) + 0 + 5 + 12 + 21 = 35$$

Arithmetic Series

Sum of n terms from an arithmetic sequence

Formula

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$a_n = \boxed{a_1 + (n-1)d}$$

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

<u>SAT</u> <u>Ex</u> Sum of 1 to 100 = ?

$1, 2, 3, 4 \ldots 100 \quad (d = 1)$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{100} = \frac{100}{2}(1 + 100)$$

$$S_{100} = 5050$$

Ex $a_1 = 1,\ a_2 = 4,\ S_{100} =$ ____

$$a_n = a_1 + (n-1)d$$
$$a_{100} = 1 + (100-1)(3)$$
$$a_{100} = 298$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{100} = \frac{100}{2}(1 + 298)$$
$$S_{100} = 14{,}950$$

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$
$$S_{100} = \frac{100}{2}\left(2(1) + (100-1)(3)\right)$$
$$S_{100} = 50(2 + 297)$$
$$S_{100} = 14{,}950$$

Sec 11-5

Geometric Series

Sum of n terms from a geometric sequence

Formula

$$S_n = \frac{a_1 - r a_n}{1-r}$$

$$a_n = \boxed{a_1 \cdot r^{n-1}}$$

$$S_n = \frac{a_1 - a_1 r^n}{1-r}$$

$a_1 = 3,\ r = 2,\ S_{10} = \_\_\_\_$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_{10} = (3)(2)^9$$

$$a_{10} = 1536$$

$$S_n = \frac{a_1 - r a_n}{1 - r}$$

$$S_n = \frac{3 - 2(1536)}{1 - 2} = \frac{-3069}{-1}$$

$$S_n = 3069$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_n = \frac{3 - 3(2)^{10}}{1 - 2}$$

$$S_n = \frac{-3069}{-1}$$

$$S_n = 3069$$

Infinite Geometric Series

<u>Ex</u> $5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8} \ldots$ $r = \frac{1}{2}$

$$\left.\begin{array}{l} S_1 = 5 \\ S_5 = 9.6875 \\ S_{20} = 9.9999 \end{array}\right\} \quad \begin{array}{l} \text{As } n \to \infty \\ S_n \to 10 \end{array}$$

The Limit

Rule

proper fraction

If $|r| < 1$ then

S_n "converges" to $\dfrac{a_1}{1-r}$

Note

Arithmetic series <u>never</u> converge

Tell whether the following geometric series converges; if so, state the limit.

1) $a_1 = 2$, $r = \frac{3}{2}$ > Does Not Converge

2) $a_1 = 15$, $r = -\frac{3}{10}$

$$S_n = \frac{a_1}{1-r} = \frac{15}{1-(-3/10)} = \frac{15}{\frac{13}{10}} = \frac{15}{1} \cdot \frac{10}{13} = \frac{150}{13} \text{ or } 11\frac{7}{13}$$