12 - 1

Combination (Review)

- A selection of items where order does not matter

- Formula:

$$\underset{\#\text{ items}}{n} C \underset{\text{arrangements}}{r} = \frac{n!}{r!(n-r)!} \quad \Big\} \; 0! = 1$$

Theoretical Probability

- The proportion of times an event occurs in the long run

- $P(\text{Tail After Coin Flip}) = .50$

long run

Sample Space

- Set of all possible outcomes of a random phenomena

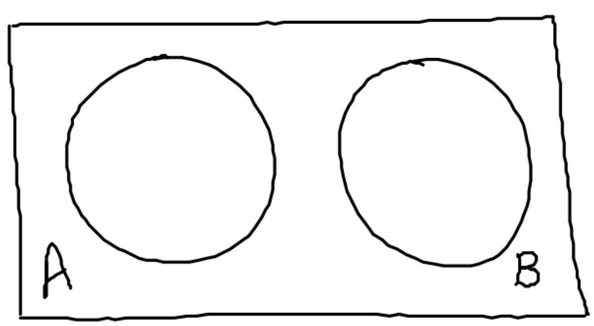
<u>Ex</u> Flip A Coin → Roll A Die (2×6 Outcomes)

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

Mutually Exclusive Events

- Events that cannot happen at the same time

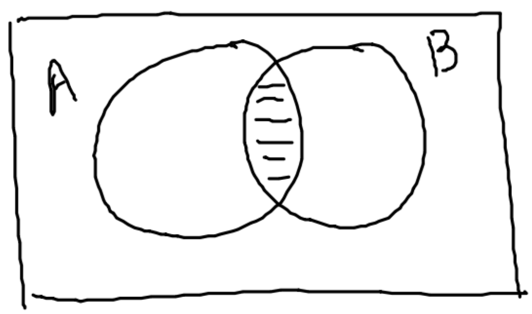
- $P(A \text{ and } B) = 0$



Independent Events

- Probability of one event has no effect on probability of another
- Knowing P(A) tells you nothing about P(B)

★ NOT mutually exclusive



Finding Probabilities

Ex Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and randomly choose a number:

$$P(\text{Prime}) = \frac{4}{9} = .4444$$

↑ 4 decimal places

$$P(\text{Multiple of } 3) = \frac{3}{9} = .3333$$

Ex Roll 2 Dice (6x6 Outcomes)

$$P(\text{Sum of } 5) = \frac{4}{36} = .1111$$

1 + 4

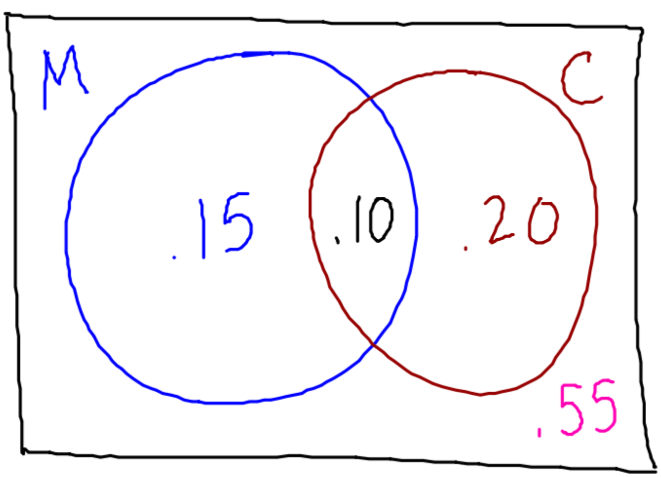
2 + 3

3 + 2

4 + 1

Venn Diagrams

In a school, 25% of teachers are math teachers, 30% are coaches and 10% are both



P(Math Only) = .15

P(Coach Only) = .20

P(Neither) = .55

Using Combinatorics

Find the probability of being dealt exactly two 7s out of 5 cards.

$$P(\text{Exactly } 2\text{-}7s) = \frac{\overset{{}_{4}C_{2}}{P(2\text{-}7s)} \text{ and } \overset{{}_{48}C_{3}}{P(3\text{-not } 7s)}}{\underset{{}_{52}C_{5}}{P(5 \text{ cards})}}$$

$$P(\text{Exactly } 2\text{-}7\text{s}) = \frac{(6)(17{,}296)}{2{,}598{,}960}$$

$$= \frac{103{,}776}{2{,}598{,}960}$$

$$= .0399$$

12-1A

Multiplication Rule

Independent

$P(A \text{ and } B) = P(A) \cdot P(B)$

Pick 2 Cards With Replacement:

$P(\text{King and Queen})$

$= P(\text{King}) \cdot P(\text{Queen})$

$= \frac{4}{52} \cdot \frac{4}{52}$

$= .0059$

Not Independent

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

Pick 2 Cards With<u>out</u> Replacement:

$P(\text{King and Queen})$

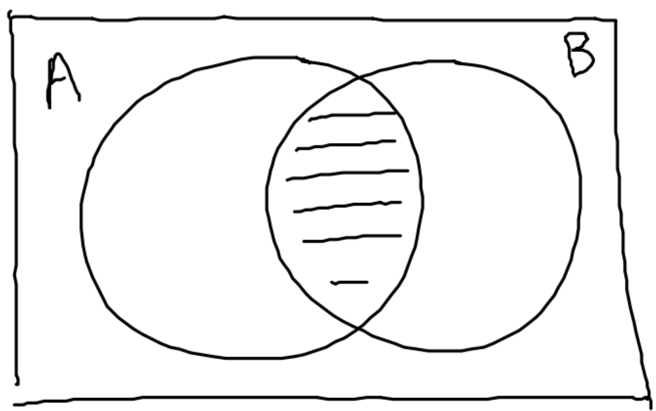
$= P(\text{King}) \cdot P(\text{Queen}|\text{King})$

$= \frac{4}{52} \cdot \frac{4}{51}$

$= .0060$

Set Notation

$$P(A \text{ and } B) = P(A \cap B)$$



Addition Rule

Mutually Exclusive

$P(A \text{ or } B) = P(A) + P(B)$

$P(\text{Student} < 17) = .62$
$P(\text{Student} > 18) = .04$

$$\boxed{\begin{aligned} &P(<17 \text{ or } >18) \\ &= P(<17) + P(>18) \\ &= .62 + .04 \\ &= .66 \end{aligned}}$$

Not Mutually Exclusive

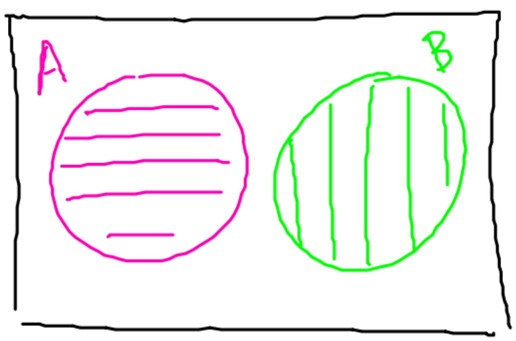
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(\text{Math Teacher}) = .25$
$P(\text{Coach}) = .30$
$P(\text{Both}) = .10$

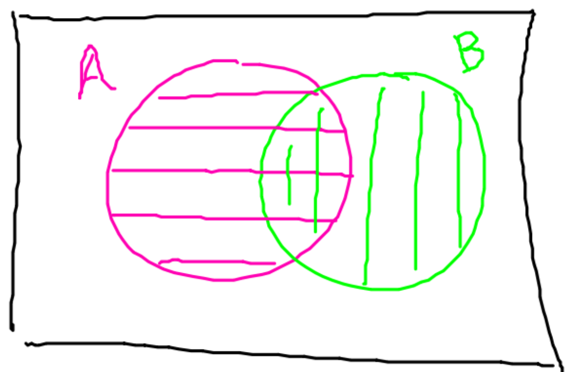
$$\boxed{\begin{aligned} &P(\text{Math or Coach}) \\ &= P(\text{Math}) + P(\text{Coach}) - P(\text{Both}) \\ &= .25 + .30 - .10 \\ &= .45 \end{aligned}}$$

Set Notation

$$P(A \text{ or } B) = P(A \cup B)$$



Mutually Exclusive



Not Mutually Exclusive

Sec 12-2

Conditional Probability

- The probability an event occurs "given that" another event has occured
- Used in multiplication rule

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

"Given That"

(If $P(B|A) = P(B)$ then A and B are independent)

Calculating Conditional Probabilities

1) Directly from problem

a) Pick 2 cards without replacement

$$P(\text{King} \mid \text{King}) = \frac{3}{51} = .0588$$

b)

	Sports	Hiking	Reading	Texting	Shopping	Other	
Female	39	48	85	62	71	29	334
Male	67	58	76	54	68	39	362
	106	106	161	116	139	68	696

$$P(\text{Sports}) = \frac{106}{696} = .1522$$

$$P(\text{Female and Sports}) = \frac{39}{696} = .0560$$

$$P(\text{Female} \mid \text{Sports}) = \frac{39}{106} = .3679$$

$$P(\text{Sports} \mid \text{Female}) = \frac{39}{334} = .1168$$

2) Use Formula (Probabilities Given)

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Ex $P(\text{Heavy Snow}) = .40$

$P(\text{Schools Close}) = .02$

$P(\text{Heavy Snow and Schools Close}) = .32$

Not Independent

$(.4)(.02) \neq .32$

Find $P(\text{Schools Close} \mid \text{Heavy Snow})$

$$= \frac{P(\text{Heavy Snow and Schools Close})}{P(\text{Heavy Snow})} = \frac{.32}{.40} = .80$$

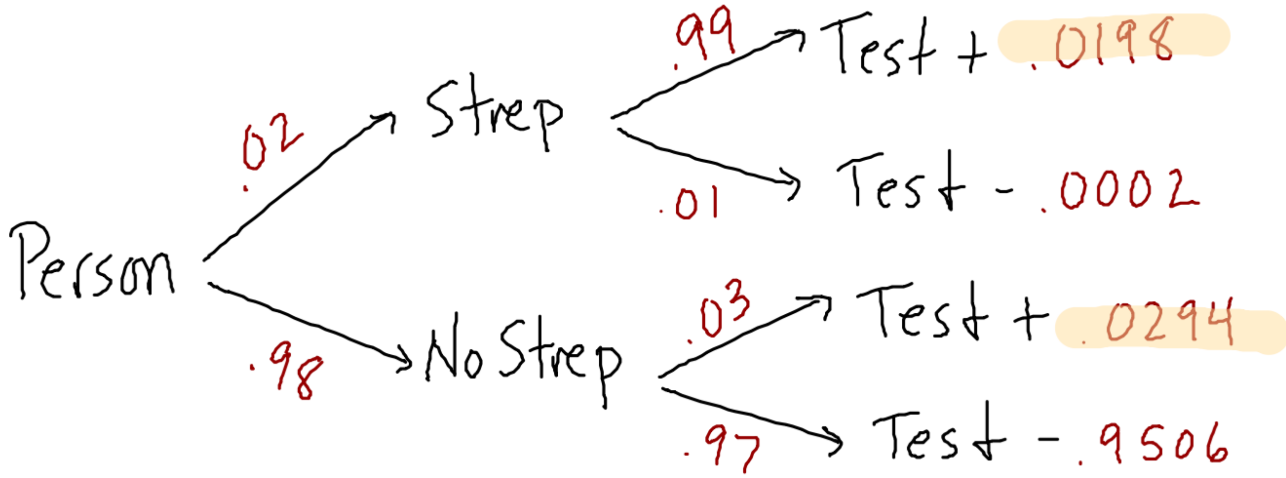
3) Use Tree Diagram / Formula

P (Strep Throat) = .02

P (Test Positive With Strep) = .99

P (Test Positive Without Strep) = .03

<u>Find</u> P(False Positive) = P (Healthy | Test Pos)



$$P(\text{Healthy} \mid \text{Test Pos}) = \frac{P(\text{Test Pos and Healthy})}{P(\text{Test Pos})}$$

$$= \frac{.0294}{.0198 + .0294}$$

$$= .5976$$