Sec 6-7

Pizza Plus offers 6 toppings: Pepperoni (P), Sausage (S), Mushroom (M), Onions (O), Green Peppers (G) and Black Olives (B). In how many ways is it possible to select 2 toppings?

PS	SM	MO	OG	GB
PM	SO	MG	OB	
PO	SG	MB		
PG	SB			
PB				

15 ways

Combination

- A selection of items where order does <u>not</u> matter

- Formula:

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} \quad \Big\} \; 0! = 1 $$

n — # items

r — # arrangements

Ex How many ways is it possible to select 2 toppings out of 6 ?

$$_6C_2 = \frac{6!}{2!(6-2)!} = \frac{6\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{2\cdot1(\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1})} = 15$$

Calculator?

Permutation

- An arrangement of items in some particular order

- Formula:

$$_nP_r = \frac{n!}{(n-r)!}$$

Ex 10 runners are in a race... how many arrangements of 1st, 2nd and 3rd-place finishes are possible?

$$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 720$$

Calculator?

Sec 6-8

Look For Patterns

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a^1 + 1b^1$$

$$(a+b)^2 = 1a^2 + 2a^1b^1 + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4$$

$$(a+b)^5 = 1a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1b^5$$

Paschal's Triangle (of Coefficients)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

↓

Combinations of Coefficients

$$
\begin{array}{cccccc}
 & & {}_0C_0 & & & \\
 & {}_1C_0 & {}_1C_1 & & & \\
{}_2C_0 & {}_2C_1 & {}_2C_2 & & & \\
{}_3C_0 & {}_3C_1 & {}_3C_2 & {}_3C_3 & & \\
{}_4C_0 & {}_4C_1 & {}_4C_2 & {}_4C_3 & {}_4C_4 & \\
{}_5C_0 & {}_5C_1 & {}_5C_2 & {}_5C_3 & {}_5C_4 & {}_5C_5
\end{array}
$$

$$
\downarrow
$$

$$
{}_nC_0 \quad {}_nC_1 \quad {}_nC_2 \quad {}_nC_3 \quad {}_nC_4 \quad \cdots \quad {}_nC_n
$$

Expand $(x-2)^4$

Patterns

$$\boxed{\begin{array}{ccccc} {}_4C_0 & {}_4C_1 & {}_4C_2 & {}_4C_3 & {}_4C_4 \\ 1a^4 & 4a^3b & 6a^2b^2 & 4ab^3 & 1b^4 \end{array}}$$

$$= 1(x)^4 \quad 4(x)^3(-2)^1 \quad 6(x)^2(-2)^2 \quad 4(x)(-2)^3 \quad (-2)^4$$

$$= 1(x^4) \quad 4(x^3)(-2) \quad 6(x^2)(4) \quad 4(x)(-8) \quad 16$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

Expand $(2x+3)^6$

$$\begin{array}{ccccccc} {}_6C_0 & {}_6C_1 & {}_6C_2 & {}_6C_3 & {}_6C_4 & {}_6C_5 & {}_6C_6 \\ 1a^6 & 6a^5b & 15a^4b^2 & 20a^3b^3 & 15a^2b^4 & 6ab^5 & 1b^6 \end{array}$$

$$= 1(2x)^6,\ 6(2x)^5(3),\ 15(2x)^4(3)^2,\ 20(2x)^3(3)^3,\ 15(2x)^2(3)^4,\ 6(2x)(3)^5,\ 1(3)^6$$

$$= (64x^6),\ 6(32x^5)(3),\ 15(16x^4)(9),\ 20(8x^3)(27),\ 15(4x^2)(81),\ 6(2x)(243),\ 729$$

$$= 64x^6 + 576x^5 + 2160x^4 + 4320x^3 + 4860x^2 + 2916x + 729$$

12 - 1

Theoretical Probability

- The proportion of times an event occurs <u>in the long run</u>

- P (Tail After Coin Flip) = .50

long run

Sample Space

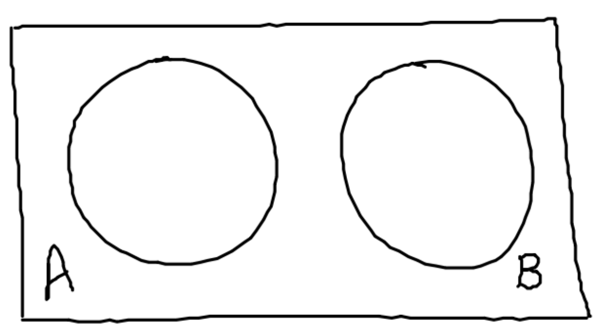
- Set of all possible outcomes of a random phenomena

<u>Ex</u> Flip A Coin $\rightarrow$ Roll A Die (2 × 3 Outcomes)

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

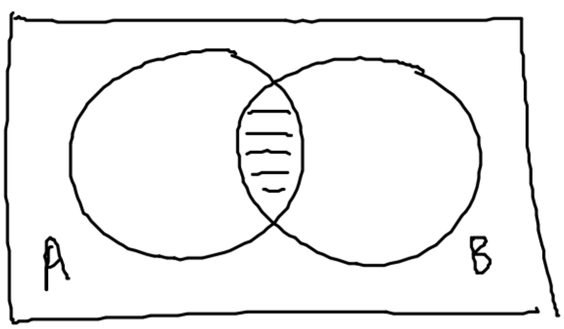
Mutually Exclusive Events

- Events that cannot happen at the same time
- $P(A \text{ and } B) = 0$



Independent Events

- Probability of one event has no effect on probability of another
- Knowing P(A) tells you nothing about P(B)
- NOT mutually exclusive



Finding Probabilities

<u>Ex</u> Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and randomly choose a number:

$$P(\text{Prime}) = \frac{4}{9} = .4444$$

Round 4 Decimal places

$$P(\text{Multiple of } 3) = \frac{3}{9} = .3333$$

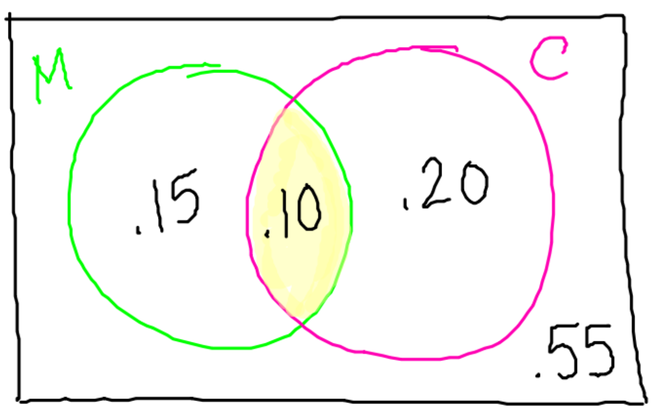
<u>Ex</u> Roll 2 Dice

$$P(\text{Sum of } 5) = \frac{4}{36} = .1111$$

6x6
outcomes

Venn Diagrams

In a school, 25% of teachers are math teachers, 30% are coaches and 10% are both



P(Math Only) = .15

P(Coach Only) = .20

P(Neither) = .55

Using Combinatorics

Find the probability of being dealt exactly two 7s out of 5 cards.

$$P(\text{Exactly } 2\text{-}7\text{s}) = \frac{\overbrace{P(2\text{-}7\text{s})}^{{}_4C_2} \text{ and } \overbrace{P(3\text{-non } 7\text{s})}^{{}_{48}C_3}}{\underbrace{P(5 \text{ cards})}_{{}_{52}C_5}}$$

$$P(\text{Exactly } 2\text{-}7\text{s}) = \frac{{}_4C_2 \cdot {}_{48}C_3}{{}_{52}C_5}$$

$$= \frac{(6)(17296)}{2{,}598{,}960}$$

$$= .0399$$

12-1A

Multiplication Rule

Independent

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Pick 2 Cards With Replacement:

$P(\text{King and Queen})$

$= P(\text{King}) \cdot P(\text{Queen})$

$= \frac{4}{52} \cdot \frac{4}{52}$

$= .0059$

Not Independent

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Pick 2 Cards Without Replacement:

$P(\text{King and Queen})$

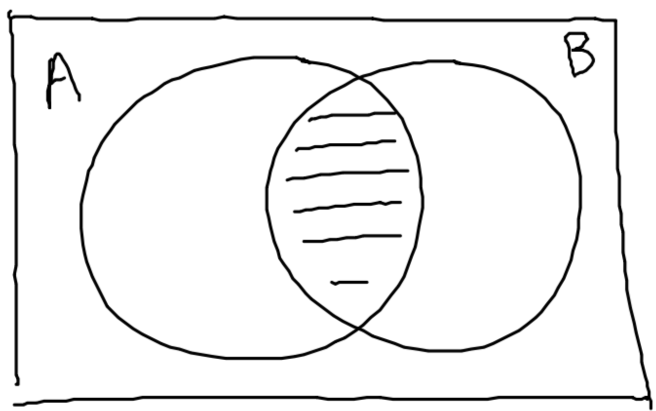
$= P(\text{King}) \cdot P(\text{Queen}|\text{King})$

$= \frac{4}{52} \cdot \frac{4}{51}$

$= .0060$

Set Notation

$$P(A \text{ and } B) = P(A \cap B)$$



Addition Rule

Mutually Exclusive

$P(A \text{ or } B) = P(A) + P(B)$

$P(\text{Student} < 17) = .62$

$P(\text{Student} > 18) = .04$

$$\boxed{\begin{aligned} &P(<17 \text{ or } >18) \\ &= P(<17) + P(>18) \\ &= .62 + .04 \\ &= .66 \end{aligned}}$$

Not Mutually Exclusive

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(\text{Math Teacher}) = .25$

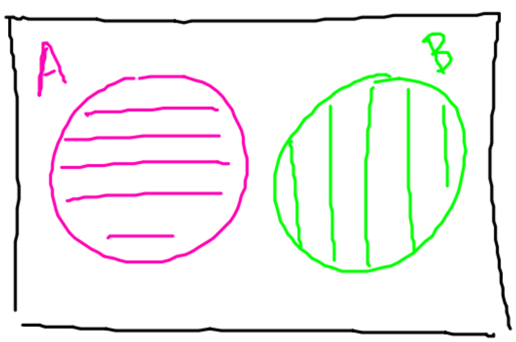
$P(\text{Coach}) = .30$

$P(\text{Both}) = .10$

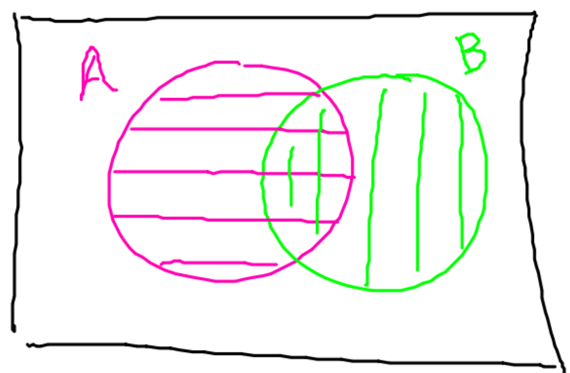
$$\boxed{\begin{aligned} &P(\text{Math or Coach}) \\ &= P(\text{Math}) + P(\text{Coach}) - P(\text{Both}) \\ &= .25 + .30 - .10 \\ &= .45 \end{aligned}}$$

Set Notation

$$P(A \text{ or } B) = P(A \cup B)$$



Mutually Exclusive



Not Mutually Exclusive

Sec 12-2

Conditional Probability

- The probability an event occurs "given that" another event has occured
- Used in multiplication rule

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

"Given That"

Calculating Conditional Probabilities

1) Directly from problem

a) Pick 2 cards <u>without</u> replacement

$$P(\text{King} \mid \text{King}) = \frac{3}{51} = .0588$$

b)

	Sports	Hiking	Reading	Texting	Shopping	Other	
Female	39	48	85	62	71	29	334
Male	67	58	76	54	68	39	362
	106	106	161	116	139	68	696

$$P(\text{Sports}) = \frac{106}{696} = .1523$$

$$P(\text{Female and Sports}) = \frac{39}{696} = .0560$$

$$P(\text{Female} \mid \text{Sports}) = \frac{39}{106} = .3679$$

$$P(\text{Sports} \mid \text{Female}) = \frac{39}{334} = .1168$$

2) Use Formula (Probabilities Given)

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Ex $P(\text{Heavy Snow}) = .4$ $\quad P(\text{Schools Close}) = .5$

$P(\text{Heavy Snow and Schools Close}) = .32$

Not Indpd

$(.4)(.5) \neq .32$

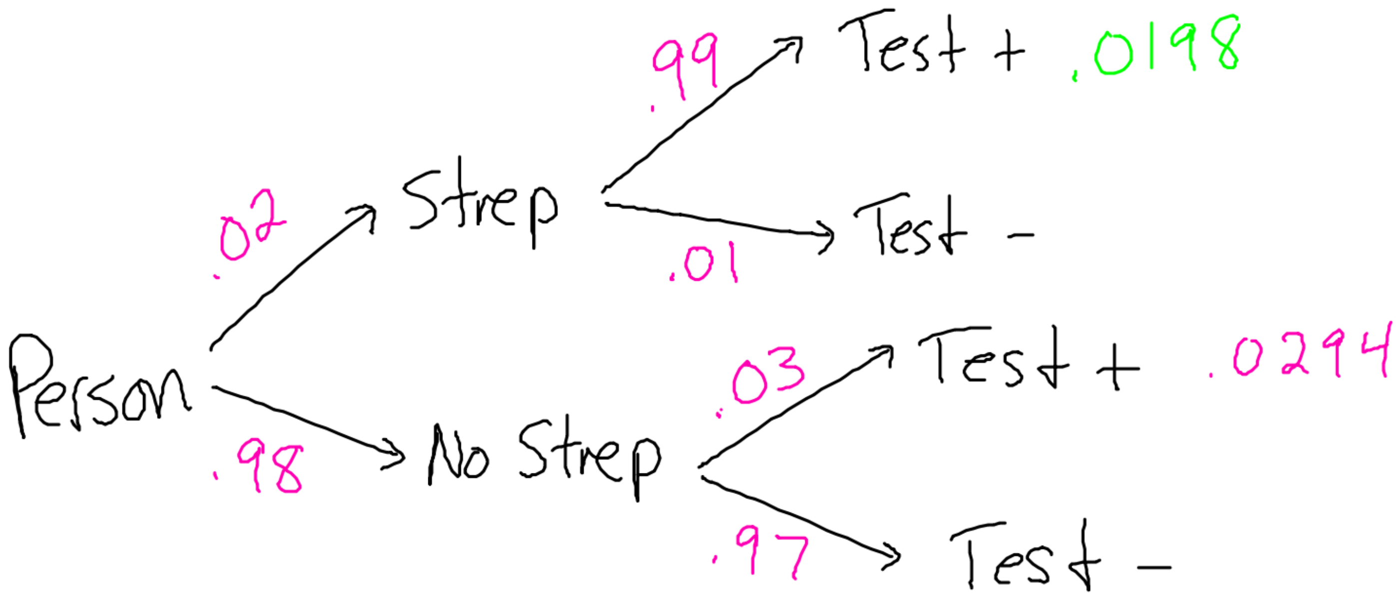
$$\boxed{P(\text{Schools Close} \mid \text{Heavy Snow}) = \frac{P(\text{Heavy Snow and Close})}{P(\text{Heavy Snow})} = \frac{.32}{.4} = .8}$$

3) Use Tree Diagrams / Formula

P(Strep) = .02   P(Test Pos w/ Step) = .99

P(Test Pos w/out Strep) = .03

Find P(False Positive) = P(Healthy | Test Pos)



$$P(\text{Healthy} \mid \text{Test Pos}) = \frac{P(\text{Test Pos and Healthy})}{P(\text{Test Pos})}$$

$$\frac{.0294}{.0198 + .0294} = \frac{.0294}{.0492} = .5976$$