3 X 3 INVERSES

$$\text{Given } M = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 2 \\ 6 & 8 & 7 \end{bmatrix} \text{ find its inverse } (M^{-1})$$

1. Evaluate the determinant of M:

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 1 & 2 \\ 6 & 8 & 7 \end{vmatrix} \begin{matrix} 2 & 3 \\ 5 & 1 \\ 6 & 8 \end{matrix} = (14 + 36 + 160) - (24 + 32 + 105) = 49$$

NOTE: If determinant = 0, there is no inverse

2. Transpose M by interchanging the rows and columns of M:

$$M^T = \begin{bmatrix} 2 & 5 & 6 \\ 3 & 1 & 8 \\ 4 & 2 & 7 \end{bmatrix}$$

3. Evaluate the minor determinant for each element of M^T:

$$\begin{bmatrix} 1 & 8 \\ 2 & 7 \end{bmatrix} = 7 - 16 = -9 \qquad \begin{bmatrix} 3 & 8 \\ 4 & 7 \end{bmatrix} = 21 - 32 = -11 \qquad \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = 6 - 4 = 2$$

$$\begin{bmatrix} 5 & 6 \\ 2 & 7 \end{bmatrix} = 35 - 12 = 23 \qquad \begin{bmatrix} 2 & 6 \\ 4 & 7 \end{bmatrix} = 14 - 24 = -10 \qquad \begin{bmatrix} 2 & 5 \\ 4 & 2 \end{bmatrix} = 4 - 20 = -16$$

$$\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} = 40 - 6 = 34 \qquad \begin{bmatrix} 2 & 6 \\ 3 & 8 \end{bmatrix} = 16 - 18 = -2 \qquad \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = 2 - 15 = -13$$

4. Insert the values for each minor determinant according to the following pattern to create the *adjoint matrix*:

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \longrightarrow \text{Adjoint Matrix} = \begin{bmatrix} -9 & 11 & 2 \\ -23 & -10 & 16 \\ 34 & 2 & -13 \end{bmatrix}$$

5. Multiply each element of the adjoint matrix by the reciprocal of the determinant:

$$M^{-1} = \frac{1}{49}\begin{bmatrix} -9 & 11 & 2 \\ -23 & -10 & 16 \\ 34 & 2 & -13 \end{bmatrix} = \begin{bmatrix} \frac{-9}{49} & \frac{4}{49} & \frac{2}{49} \\ \frac{-23}{49} & \frac{-10}{49} & \frac{16}{49} \\ \frac{34}{49} & \frac{2}{49} & \frac{-13}{49} \end{bmatrix}$$