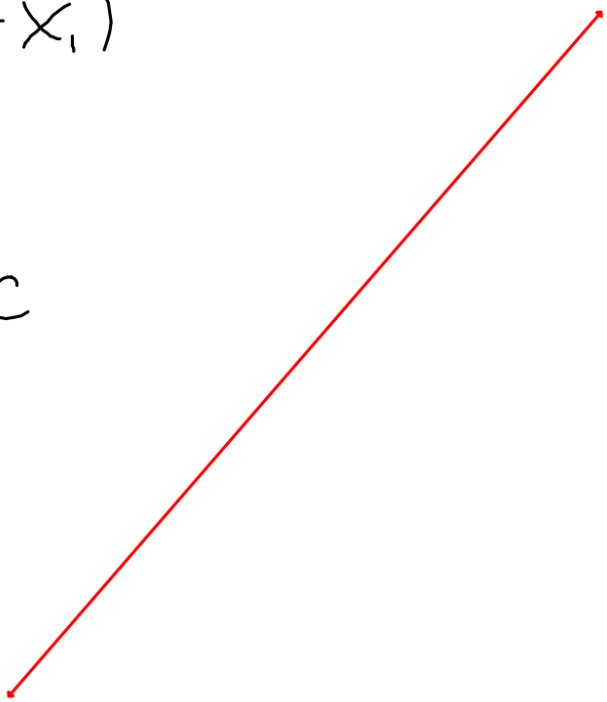
Secs 5-2 and 5-3

Graphs of Equations (Solutions)

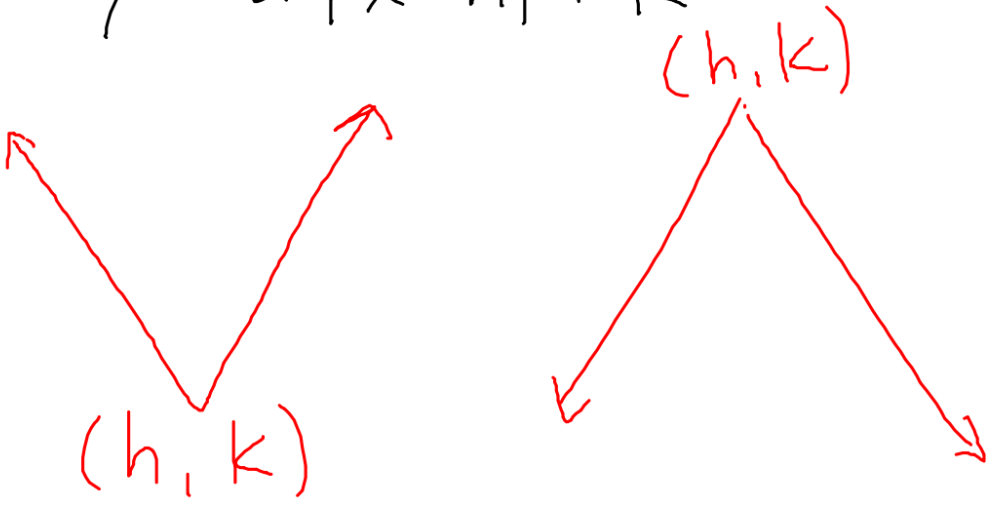
1) $y - y_1 = m(x - x_1)$

$y = mx + b$

$Ax + By = C$



2) $y = a|x-h| + k$



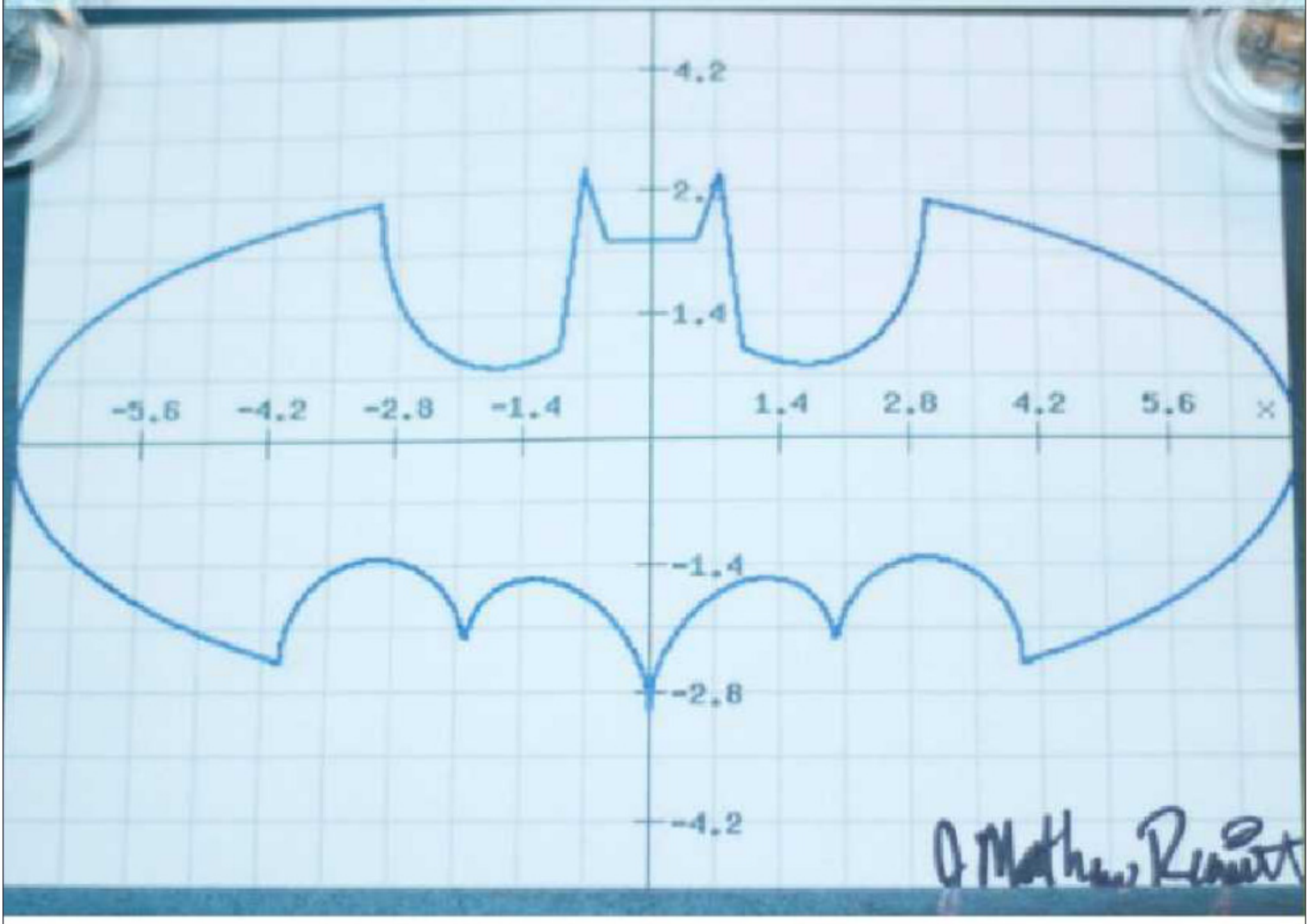
3) Internet Equation ...

Batman Equation

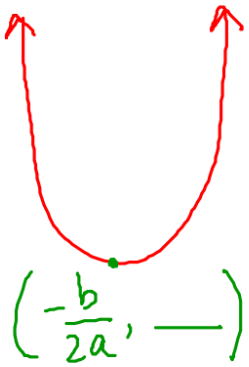
$$\left(\left(\frac{x}{7}\right)^2\sqrt{\frac{|x|-3}{|x|-3}}+\left(\frac{y}{3}\right)^2\sqrt{\frac{\left|y+\frac{3\sqrt{33}}{7}\right|}{y+\frac{3\sqrt{33}}{7}}}-1\right)\cdot\left(\left|\frac{x}{2}\right|-\left(\frac{3\sqrt{33}-7}{112}\right)x^2-3+\sqrt{1-\left(\left||x|-2\right|-1\right)^2}-y\right)$$

$$\cdot\left(9\sqrt{\frac{|(|x|-1)(|x|-.75)|}{(1-|x|)(|x|-.75)}}-8|x|-y\right)\cdot\left(3|x|+.75\sqrt{\frac{|(|x|-.75)(|x|-.5)|}{(.75-|x|)(|x|-.5)}}-y\right)$$

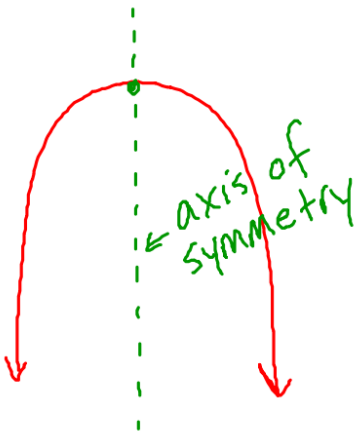
$$\cdot\left(2.25\sqrt{\frac{|(x-.5)(x+.5)|}{(.5-x)(.5+x)}}-y\right)\cdot\left(\frac{6\sqrt{10}}{7}+(1.5-.5|x|)\sqrt{\frac{|x|-1}{|x|-1}}-\frac{6\sqrt{10}}{14}\sqrt{4-(|x|-1)^2}-y\right)=0$$



4) $y = ax^2 + bx + c$

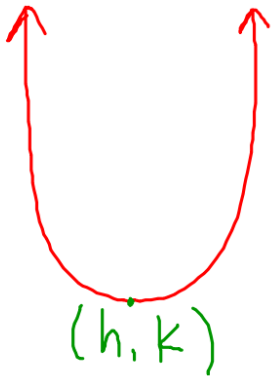


$a > 0$

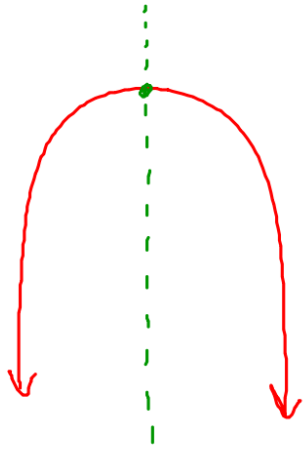


$a < 0$

$y = a(x-h)^2 + k$

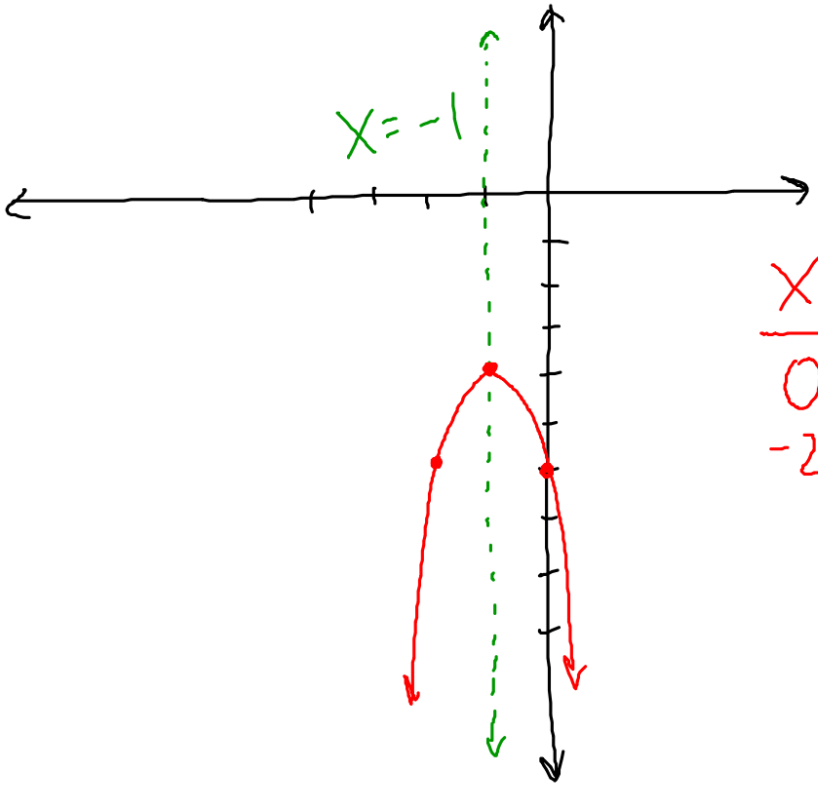


$a > 0$



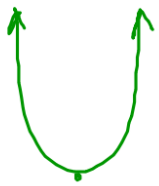
$a < 0$

Graph $y = -2(x+1)^2 - 4$

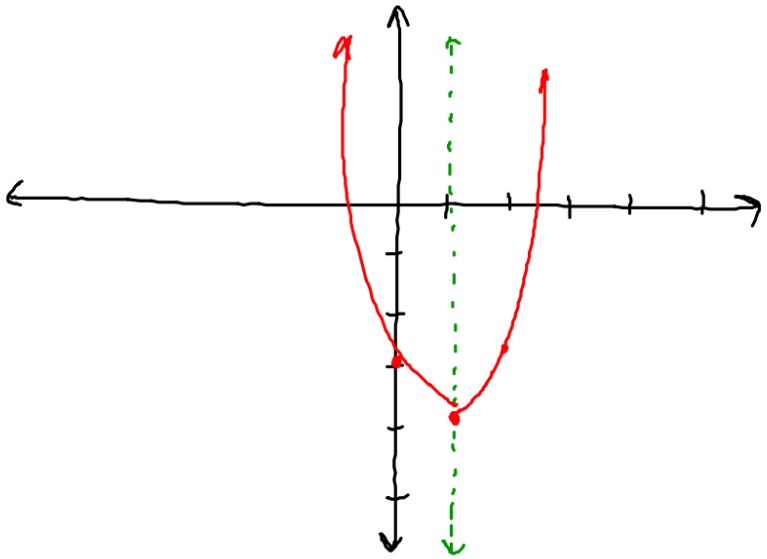


x	y
0	-6
-2	-6

Graph $y = x^2 - 2x - 3$



Vertex $\left(-\frac{b}{2a}, \_\_\right) = \left(-\frac{-2}{2(1)}, \_\_\right) = (1, -4)$



X	Y
0	-3
2	-3

Write the equation of a parabola in vertex form: Vertex $(3,5)$ (h, k), Point $(1,-2)$ (x, y)

$$y = a(x-h)^2 + k$$

$$-2 = a(1-3)^2 + 5$$

$$-2 = 4a + 5$$

$$a = -\frac{7}{4}$$

$$\boxed{y = -\frac{7}{4}(x-3)^2 + 5}$$

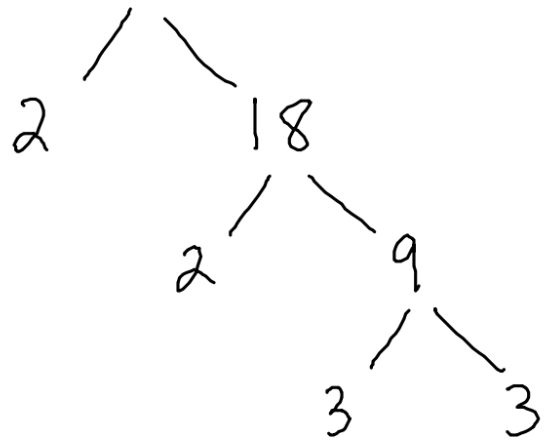
Sec 5-4

Prime Numbers

- Exactly 2 factors (1 and itself)

$P = \{2, 3, 5, 7, 11 \ldots\}$

Prime Factor 36



$36 = (2)(2)(3)(3) = 36$ ✓

Common Factors

$$5x^2 - 10x = 5x(x - 2)$$

Difference of Squares (Binomials)

$x^2 - 4 = (x-2)(x+2)$ > FOIL

$4x^2 - 25 = (2x-5)(2x+5)$

conjugates

Quadratic Trinomials $(a=1)$

$$x^2+10x+9 = (x+9)(x+1)$$

$$= (x+1)(x+9)$$

Quadratic Trinomials ($a > 1$)

a) Guess and Check

$$2x^2 + 9x - 5 = (2x - 1)(x + 5)$$

F O I L

$$(2x + 1)(x - 5)$$

$$(2x - 5)(x + 1)$$

$$(2x + 5)(x - 1)$$

b) Borrow / Payback

$2x^2 + 9x - 5 = x^2 + 9x - 10$

$ac = -10$

$= (x - \frac{1}{2})(x + \frac{10}{2})$

$= (2x - 1)(x + 5)$

c) Split Middle Term

$2x^2 + 9x - 5$

$ac = -10$

$(-1)(10) = -10$

$(-1) + (10) = 9$

$= (2x^2 - 1x) + (10x - 5)$

$= x(2x-1) + 5(2x-1)$

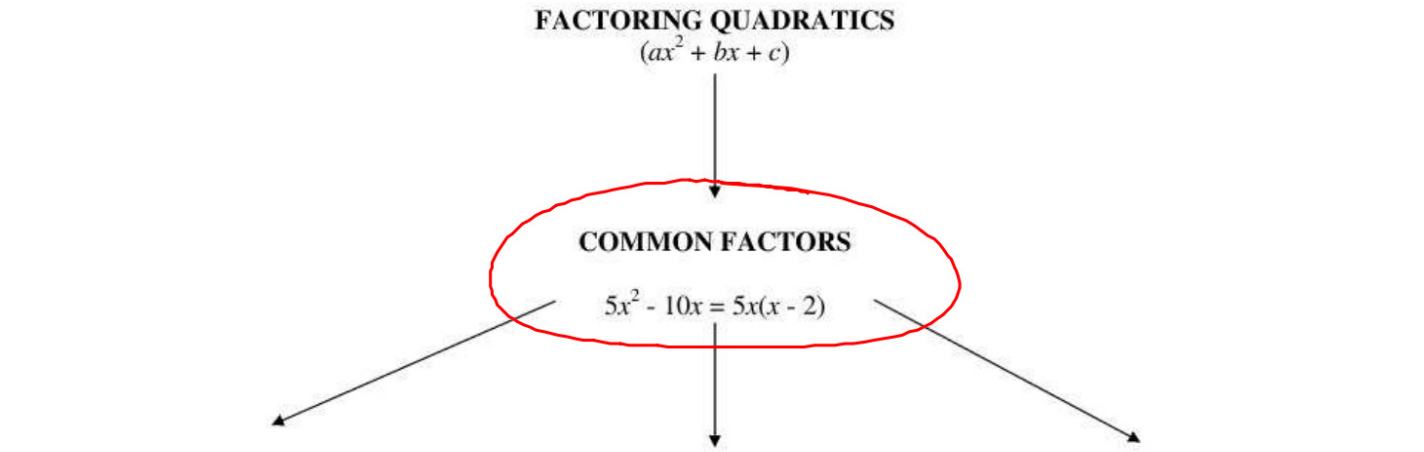
$= x(\quad) + 5(\quad)$

$= (\quad)(x + 5)$

$= (2x-1)(x+5)$

Factoring Techniques (Review)

- Handout



DIFFERENCE OF SQUARES

$x^2 - 4 = (x - 2)(x + 2)$

Conjugates

QUADRATIC TRINOMIALS (a = 1)

$x^2 + 10x + 9 = (x + 1)(x + 9)$

Determine 2 numbers that multiply to get c and add to get b

QUADRATIC TRINOMIALS (a >1)

$2x^2 + 9x - 5 = (2x - 1)(x + 5)$

* Guess and check (FOIL)
* Split the middle term
* Borrow/payback

Always check answer with distributive property or FOIL

Factoring By Grouping (4 Terms)

$$(7x^3 + 21x^2) + (8x + 24)$$

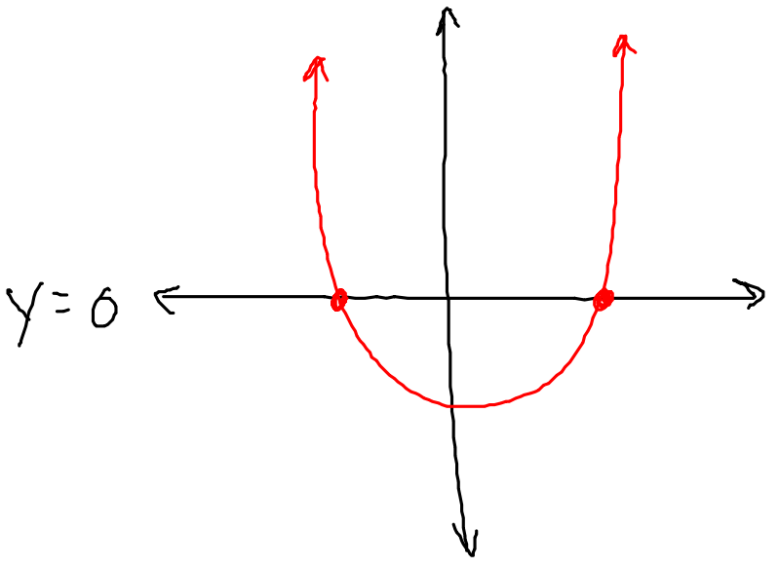
$$= 7x^2(x + 3) + 8(x + 3)$$

$$= (x+3)(7x^2+8)$$

Sec 5-5

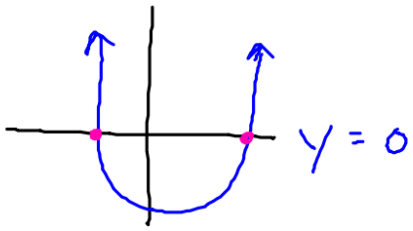
Parabolas → Quadratic Equations

$y = ax^2 + bx + c$ $\quad$ $0 = ax^2 + bx + c$



Solving Quadratic Equations $(ax^2 + bx + c = 0)$

1) Graphing



★ 2) Factoring

3) Use $\sqrt{\ }$ (if $b = 0$) ?

4) Complete The Square

5) Quadratic Formula

Solve By Factoring

$$x^2 + x = 6$$

$$x^2 + x - 6 = 0 \qquad \text{Set} = 0\ ?$$

$$(x+3)(x-2) = 0 \qquad \text{Factor}$$

$$x+3=0 \ \underline{\text{or}} \ x-2=0 \qquad \text{Set Factors} = 0$$

$$x = -3 \qquad x = 2 \qquad \text{Solve}$$

Solve By Taking Square Roots ($b = 0$)

$$x^2 - 9 = 0$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

$$\sqrt{(x-4)^2} = \sqrt{25}$$

$$\sqrt{\text{☺}^2} = \sqrt{25}$$

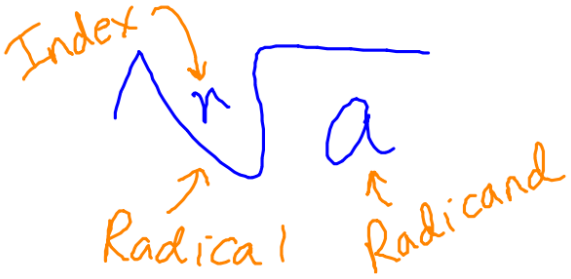
$$\text{☺} = \pm 5$$

$$x - 4 \overset{+4}{} = \pm 5 \overset{+4}{}$$

$$x = 5 + 4 \text{ or } -5 + 4$$

$$x = 9 \text{ or } -1$$

Simplifying Radicals



1) No perfect square factors (4, 9, 16, 25, 36...) in radicand

$$\ldots \sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$$

$$\ldots \sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$$

2) No radicals in denominators

$$\ldots \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$$

Solving By Taking Square Roots

$$\sqrt{(x+3)^2} = \sqrt{12}$$

OTTSS

$$x + 3 = \pm 2\sqrt{3}$$

$$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$$

$$x = -3 \pm 2\sqrt{3}$$

Sec 5-6

Imaginary Numbers

$$\sqrt{-1} = \boxed{i}$$

$$i^2 = \boxed{-1}$$

$$i^3 = (i \cdot i) \cdot i = (i^2)\, i = \boxed{-i}$$

$$i^4 = (i \cdot i) \cdot (i \cdot i) = (i^2)(i^2) = \boxed{1}$$

Simplify

$$\sqrt{-4} = \sqrt{-1}\sqrt{4} = i2 = \boxed{2i}$$

$$\sqrt{-45} = \sqrt{-1}\sqrt{9}\sqrt{5} = i3\sqrt{5} = \boxed{3i\sqrt{5}}$$

$$i^{15} = i^{3} = \boxed{-i}$$

$$4\overline{)\begin{array}{r}3\\15\\12\\\hline 3\end{array}}$$

Complex Numbers

$$a + bi$$

a: real

bi: imaginary

Simplify

Ex $(9+10i)-(7+4i) = \boxed{2+6i}$

Ex $(5+2i)(7-4i)$ > FOIL

$= 35 - 20i + 14i - \cancel{8i^2}$

$+8$

$= \boxed{43-6i}$

Solve {Complex}

$x^2 - 9 = 0$

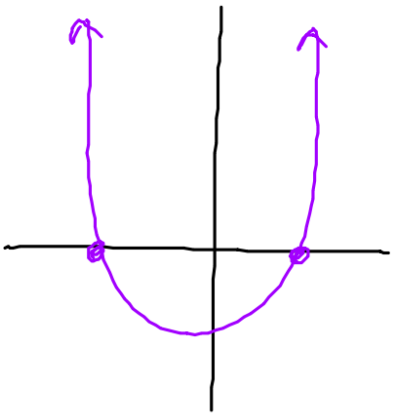
$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

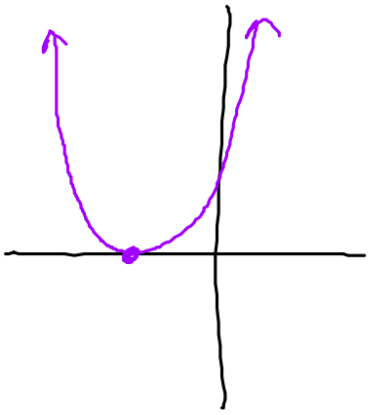
$x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

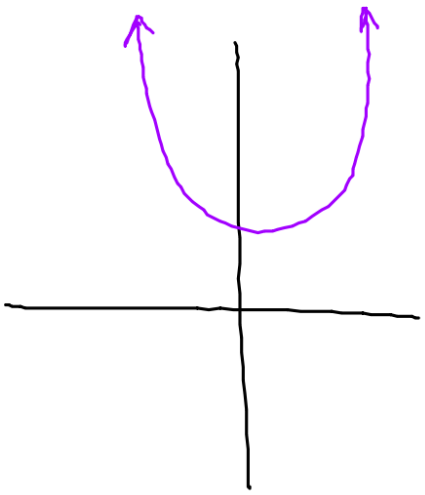
$x = \pm 3i$



2 Real Solutions



1 Real Solution



2 Imaginary Solutions

Sec 5-7

Complete The Square

$$x^2+6x+\underline{\quad 9 \quad} = (x+3)(x+3) = (x+3)^2$$

$$\left(\frac{6}{2}\right)^2$$

$$x^2+5x+\underline{\quad \frac{25}{4} \quad} = \left(x+\frac{5}{2}\right)\left(x+\frac{5}{2}\right) = \left(x+\frac{5}{2}\right)^2$$

$$\left(\frac{5}{2}\right)^2$$

Solve By Completing The Square

$$x^2 + 2x - 5 = 0$$

$$x^2 + 2x + \underline{1} = 5 \overset{+1}{}$$

$$\sqrt{(x+1)^2} = \sqrt{6}$$

$$x + 1 = \pm\sqrt{6}$$

$$x = -1 \pm \sqrt{6}$$

$$x^2 + 4x + 5 = 0$$

$$x^2 + 4x + \underline{4} = -5 \overset{+4}{}$$

$$\sqrt{(x+2)^2} = \sqrt{-1}$$

$$x + 2 = \pm i$$

$$x = -2 \pm i$$

$$\frac{4x^2}{4} + \frac{10x}{4} + \frac{7}{4} = \frac{0}{4} \qquad (a > 1)$$

$$x^2 + \frac{5}{2}x + \frac{7}{4} = 0$$

$$x^2 + \frac{5}{2}x + \frac{25}{16} = -\frac{28}{16} + \frac{25}{16}$$

$$\sqrt{\left(x + \frac{5}{4}\right)^2} = \sqrt{\frac{-3}{16}}$$

$$x + \frac{5}{4} = \pm \frac{i\sqrt{3}}{4}$$

$$x = \frac{-5}{4} \pm \frac{i\sqrt{3}}{4} \quad \text{or} \quad \frac{-5 \pm i\sqrt{3}}{4}$$

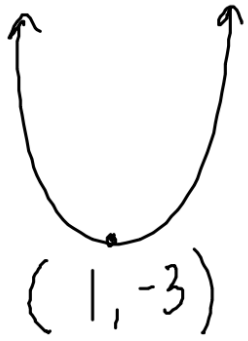
Standard Form → Vertex Form

$$y \overset{+2}{} = x^2 - 2x - 2 \overset{+2}{}$$

$$y + 2 \overset{+1}{} = x^2 - 2x + \underline{\quad 1 \quad}$$

$$y + 3 = (x-1)^2$$

$$y = (x-1)^2 - 3$$



Sec 5-8

Solve For x:

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2$

$\left(\frac{b}{2a}\right)^2$

$\frac{b^2}{4a^2}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-4ac}{4a \cdot a} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Quad Formula

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

> Discriminant

Solve: $2x^2 = -6x - 7$

$$2x^2 + 6x + 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-6 \pm \sqrt{36-4(2)(7)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{-20}}{4} = \frac{-6 \pm 2i\sqrt{5}}{4}$$

$\sqrt{-20} = \sqrt{-1}\sqrt{4}\sqrt{5}$

$$x = \frac{-6 \pm 2i\sqrt{5}}{4} = \frac{2(-3 \pm i\sqrt{5})}{4\,2} = \frac{-3 \pm i\sqrt{5}}{2}$$

Discriminant Test

$b^2 - 4ac$

< 0
2 Imaginary Solutions

$= 0$
Exactly 1 Real Solution

> 0
2 Real Solutions

↓

Perfect Square

↓

Factors

Evaluate the discriminant; tell how many real/imaginary solutions there are.

$$x^2 = 8x - 16 \rightarrow x^2 - 8x + 16 = 0$$

$$b^2 - 4ac = (-8)^2 - 4(1)(16)$$

$$= 64 - 64$$

$$= 0$$

$$\downarrow$$

Exactly one Real Solution