

Sec 6-7

Pizza Plus offers 6 toppings:
Pepperoni (P), Sausage (S), Mushroom (M),
Onions (O), Green Peppers (G) and Black
Olives (B). In how many ways is it
possible to select 2 toppings?

PS	SM	MO	OG	GB
PM	SO	MG	OB	
PO	SG	MB		
PG	SB			
PB				

} 15 ways

Combination

- A selection of items where order does not matter

Formula:

$${}^nC_r = \frac{n!}{r!(n-r)!} \quad \left. \vphantom{{}^nC_r} \right\} 0! = 1$$

$\uparrow$ $\uparrow$
#items arrangements

Ex How many ways is it possible to select 2 toppings out of 6?

$$\begin{aligned} {}_6C_2 &= \frac{6!}{2! \underset{4}{(6-2)}!} = \frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{2 \cdot 1 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\ &= \frac{30}{2} \\ &= 15 \end{aligned}$$

↑
Calculator?

Permutation

- An arrangement of items in some particular order

Formula:

$${}_n P_r = \frac{n!}{(n-r)!}$$

Ex 10 runners are in a race ... how many arrangements of 1st, 2nd and 3rd-place finishes are possible?

$${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdots \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdots \cancel{1}}$$
$$= 720$$

↑
Calculator?

Sec 6-8

Look For Patterns

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a^1 + 1b^1$$

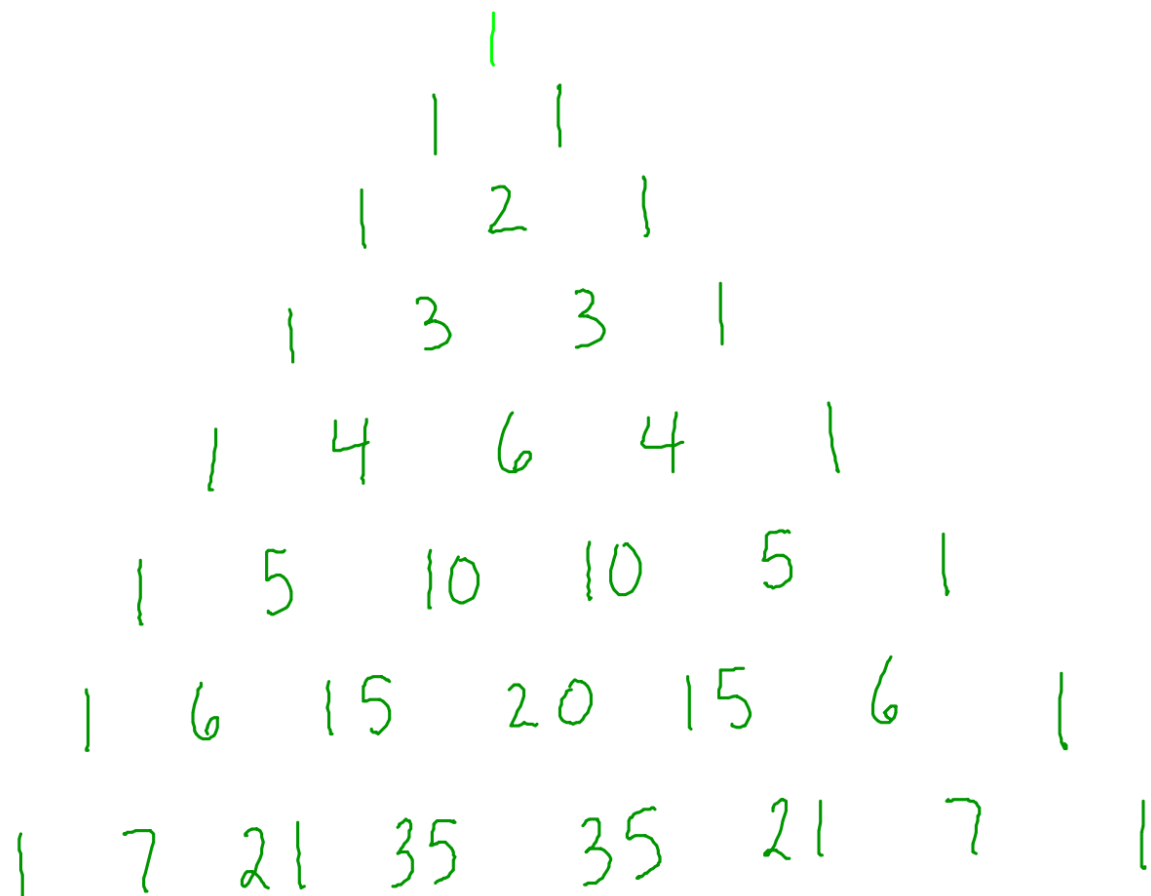
$$(a+b)^2 = 1a^2 + 2a^1b^1 + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4$$

$$(a+b)^5 = 1a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1b^5$$

Pascal's Triangle (of Coefficients)



Combinations of Coefficients

$$\begin{array}{ccccccccc} & & & & {}_0C_0 & & & & \\ & & & & {}_1C_0 & {}_1C_1 & & & \\ & & & {}_2C_0 & {}_2C_1 & {}_2C_2 & & & \\ & & {}_3C_0 & {}_3C_1 & {}_3C_2 & {}_3C_3 & & & \\ & {}_4C_0 & {}_4C_1 & {}_4C_2 & {}_4C_3 & {}_4C_4 & & & \end{array}$$

{

$$(a+b)^n = {}_nC_0 \quad {}_nC_1 \quad {}_nC_2 \quad {}_nC_3 \quad {}_nC_4 \quad \dots \quad {}_nC_n$$

Expand $(x-2)^4$

$$\begin{array}{ccccc} {}^4C_4 a^4 & {}^4C_3 a^3 b & {}^4C_2 a^2 b^2 & {}^4C_1 a b^3 & {}^4C_0 b^4 \\ 1 a^4 & 4 a^3 b & 6 a^2 b^2 & 4 a b^3 & 1 b^4 \end{array}$$

Let $a=x$ and $b=-2$:

$$= 1(x)^4 + 4(x)^3(-2) + 6(x)^2(-2)^2 + 4(x)(-2)^3 + 1(-2)^4$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

Find 4th Term of $(2x+3)^6$

$$\begin{aligned} & \uparrow \\ & {}^6C_3 a^3 b^3 \\ & 20 (2x)^3 (3)^3 \\ & 20 (8x^3) (27) \\ & 4320x^3 \end{aligned}$$