6-1 and 6-2

Monomial

Single term (3, x, $4x^2$, $\frac{1}{2}xy$ etc)

Polynomial

"Many" terms added/subtracted

Binomial x^2+4, $3x^3-2x$ etc

Trinomial $2x^2+3x-4$ etc

Naming Polynomials

By Number of Terms

By Degree (Largest exponent)

Polynomial	Degree	Name (By Degree)
$P(x) = 6x^0$	0	Constant
$P(x) = 6x^1 + 3$	1	Linear
$P(x) = 6x^2 + 3$	2	Quadratic
$P(x) = 6x^3 + 3$	3	Cubic
$P(x) = 6x^4 + 3$	4	Quartic
$P(x) = 6x^5 + 3$	5	Quintic

Ex Write a cubic trinomial

$P(x) = 5x^3 + 2x^2 + 3x$

$P(x) = 3x^3 + 4x + 2$ etc

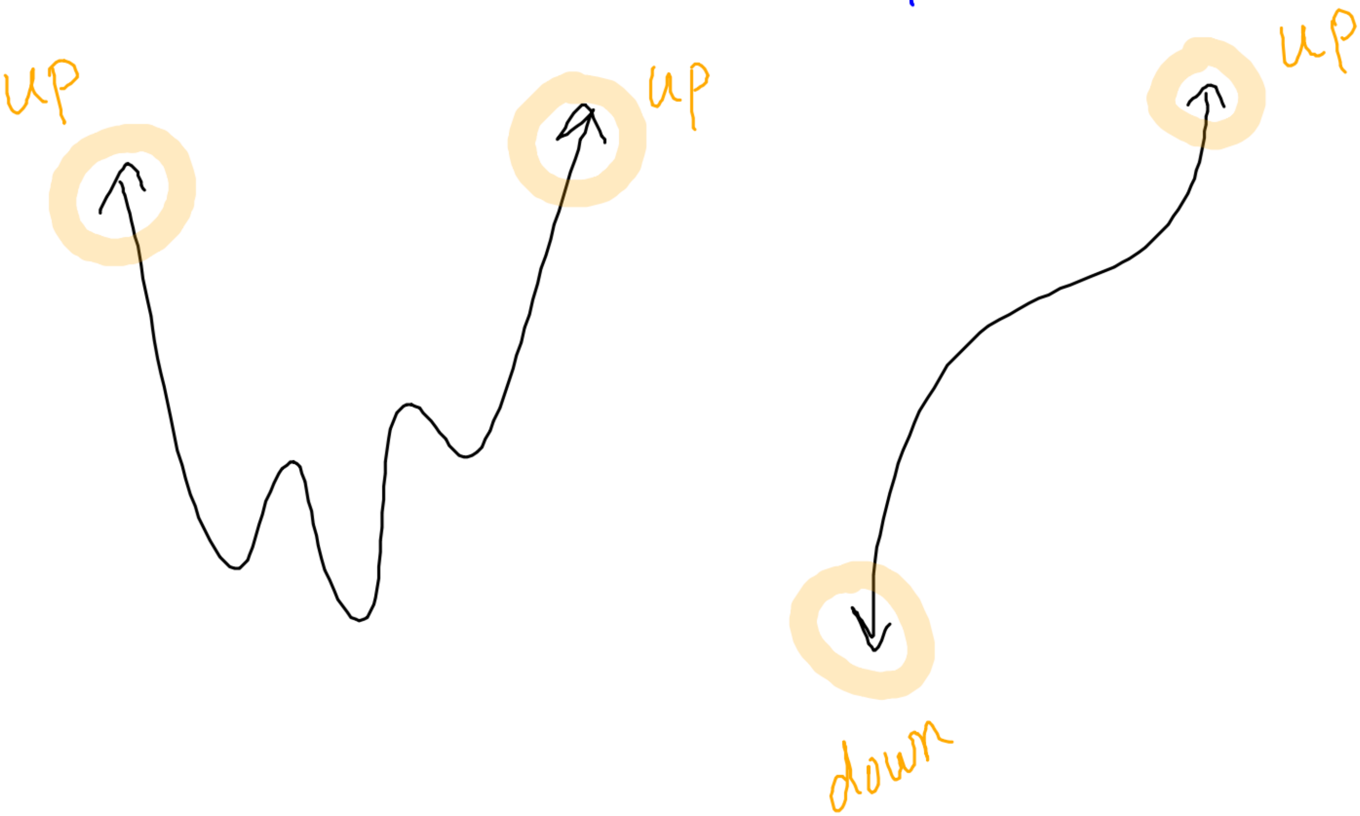
Standard Form

$$P(x) = 3x + x^2 - 4 + 2x^3$$

$$\downarrow$$

$$P(x) = 2x^3 + x^2 + 3x - 4$$

End Behaviors of Graphs



Determining End Behaviors (By Inspection)

n odd, left opposite of right (↙↗ or ↖↘)

n even, left same as right (↖↗ or ↙↘)

$$P(x) = ax^n + \cdots$$ } Standard Form

$a > 0$, Right End ↗

$a < 0$, Right End ↘

$P(x) = 2x^1$

$P(x) = x^2$

$P(x) = -2x^1$

$P(x) = -3x^2$

$P(x) = x^3$

$P(x) = x^4$

$P(x) = -4x^5$

$P(x) = -5x^4$

Factor Theorem

If $P(a) = 0$

then $(x - a)$ is a factor of $P(x)$

Ex

Let $P(x) = x^3 - 4x^2 - 3x + 18$

Find $P(-2) = (-2)^3 - 4(-2)^2 - 3(-2) + 18$ = 

$-8 \quad -16 \quad +6 \quad +18$

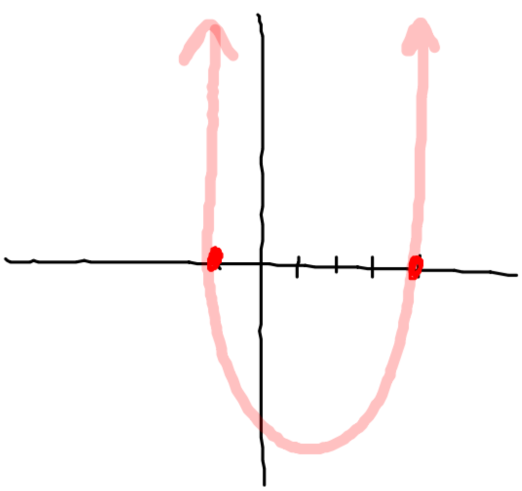
↓

Then $(x + 2)$ is a factor of $P(x)$

Find all zeros; sketch the graph.

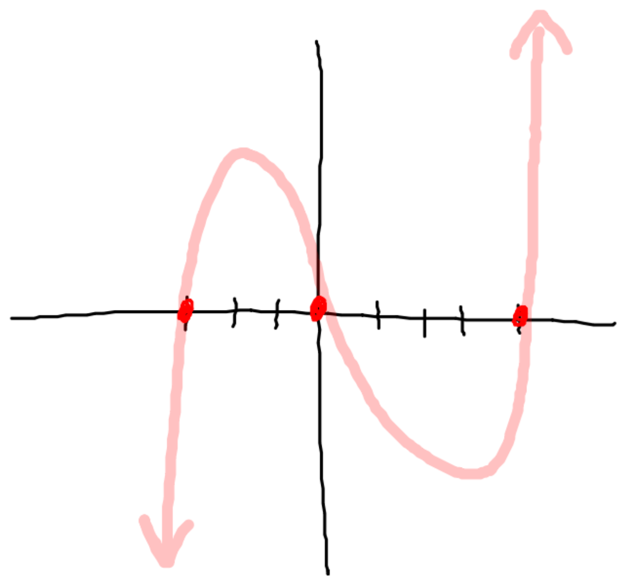
$P(x) = (x+1)(x-4)$

zeros = -1 and 4



$P(x) = 2x(x-4)(x+3)$

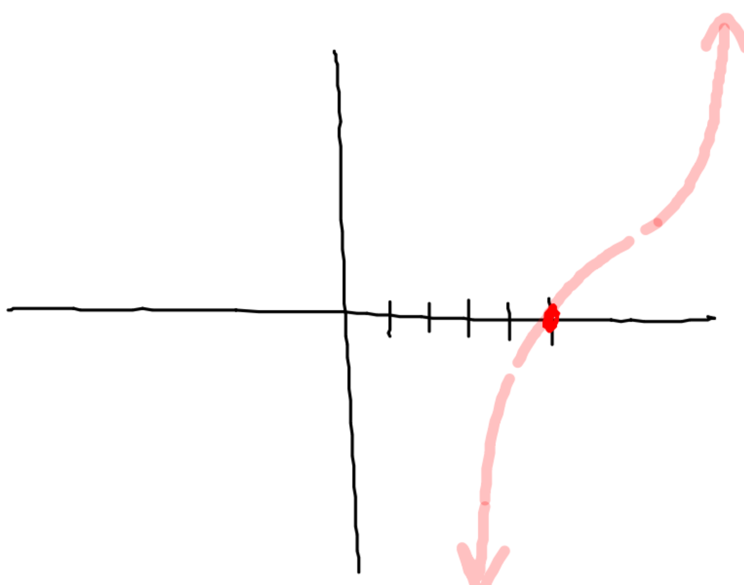
zeros = 0, 4 and -3



Find all zeros; sketch the graph.

$$P(x) = (x-5)^3 = (x-5)(x-5)(x-5)$$

zeros = 5 (Multipicity of 3)



Write a Polynomial With Zeros 1, -1 and 4

$P(x) = (x-1)[(x+1)(x-4)]$ > Write as factors

$= (x-1)(x^2-3x-4)$ > FOIL any pair

I) $P(x) = x^3 - 3x^2 - 4x - x^2 + 3x + 4$

$P(x) = x^3 - 4x^2 - x + 4$

II)

	x^2	$-3x$	-4
x	x^3	$-3x^2$	$-4x$
-1	$-x^2$	$+3x$	$+4$

$P(x) = x^3 - 4x^2 - x + 4$

III)

$$\begin{array}{r} x^2 - 3x - 4 \\ x - 1 \\ \hline -x^2 + 3x + 4 \\ x^3 - 3x^2 - 4x \quad\quad \\ \hline x^3 - 4x^2 - x + 4 \end{array}$$

Sec 6-3

Vocabulary

Quotient

Divisor | Dividend

Divide 581 by 23:

$$25\frac{6}{23}$$

$$
\begin{array}{r|l}
23 & 581 \\
 & -46 \downarrow \\
 & \overline{\;\;121} \\
 & -115 \\
 & \overline{\;\;\;\;\;\;6}
\end{array}
$$

6 ← If R = 0 then the divisor is a factor of dividend

Divide $(x^3 - 7x^2 + 15x - 9)$ by $(x - 3)$:

$$
\begin{array}{r|l}
 & x^2 - 4x + 3 \\
\hline
x - 3 & x^3 - 7x^2 + 15x - 9 \\
 & -x^3 + 3x^2 \\
\hline
 & \quad -4x^2 + 15x \\
 & \quad +4x^2 - 12x \\
\hline
 & \qquad 3x - 9 \\
 & \qquad -3x + 9 \\
\hline
 & \qquad\quad 0^{*}
\end{array}
$$

* Factor of $x^3 - 7x^2 + 15x - 9$

Divide $(x^3 - 8)$ by $(x - 2)$:

$$
\begin{array}{r|l}
 & x^2 + 2x + 4 \\
\hline
x - 2 & x^3 + 0x^2 + 0x - 8 \\
 & -x^3 + 2x^2 \\
\cline{2-2}
 & \quad 2x^2 + 0x \\
 & \quad -2x^2 + 4x \\
\cline{2-2}
 & \qquad 4x - 8 \\
 & \qquad -4x + 8 \\
\cline{2-2}
 & \qquad\quad 0
\end{array}
$$

Synthetic Division / Substitution

Divide $(x^3 - 7x^2 + 15x - 9)$ by $(x - 3)$

3	1	-7	15	-9	} Coefficients of Dividend
	↓	3	-12	9	
	1	-4	3	0	← Remainder

Coefficients of Quotient!

$x^2 - 4x + 3$

Divide $x^3 - 8$ by $(x-2)$

2	1	0	0	−8
	↓	2	4	8
	1	2	4	0 ← Remainder

↓

$x^2 + 2x + 4$

If $P(x) = 2x^4 + 6x^3 - 5x^2 - 60$, find $P(-1)$:

1) Plug 'n Chug

$$P(-1) = 2(-1)^4 + 6(-1)^3 - 5(-1)^2 - 60 = \boxed{-69}$$

2   -6   -5   -60

2) Use Synthetic Substituion

-1	2	6	-5	0	-60
	↓	-2	-4	9	-9
	2	4	-9	9	-69

-69 $= P(-1)$

Remainder Theorem

If $P(x)$ is divided by $(x-a)$

then $P(a) =$ remainder

Factor Theorem (Review)

If $P(a) = 0$

then $(x-a)$ is a factor of $P(x)$

Sec 6-4

Review

$$x^2 - 4 = (x+2)(x-2)$$ ← conjugates

$$x^2 + 4 = 1\,(x^2+4) \rightarrow \text{Prime}$$

Perfect Cubes = $\{1, 8, 27, 64, 125 \ldots\}$

Factoring Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

S O AP

$$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$

Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

S O AP

$$8x^3 + 125 = (2x + 5)(4x^2 - 10x + 25)$$

Solve Polynomial Equations
(Factor And/Or Use Quad Formula)

Ex $8x^3 - 27 = 0$

$(2x-3)(4x^2+6x+9) = 0$

↙ ↘

$2x - 3 = 0$ or $4x^2 + 6x + 9 = 0$

$2x = 3$

$x = 3/2$

$$x = \frac{-6 \pm \sqrt{36 - 4(4)(9)}}{2(4)}$$

$$= \frac{-6 \pm \sqrt{-108}}{8} = \frac{-6 \pm i6\sqrt{3}}{8} = \frac{-3 \pm 3i\sqrt{3}}{4}$$

Ex $x^4 + x^2 - 20 = 0$

Think

$x^2 + x - 20 = 0$

$(x^2 - 4)(x^2 + 5) = 0$

$x^2 - 4 = 0$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm i\sqrt{5}$

Descartes Rule of Signs

Used to determine pos<u>sib</u>le number of positive and negative real roots of P(x)

↑ solutions

$$P(x) = x^5 - 2x^4 - 3x^3 + 6x^2 - 4x + 8$$

Possible Number Of Positive Real Roots $= 4, 2$ or 0

$$P(-1) = (-1)^5 - 2(-1)^4 - 3(-1)^3 + 6(-1)^2 - 4(-1) + 8$$

$-\quad -\quad +\quad +\quad +\quad +$

Possible Number Of Negative Real Roots $= 1$

Secs 6-5 and 6-6

Where We've Been ...

Divide $P(x)$ by $(x-a)$

- Remainder
- No Remainder
  - $(x-a)$ is a factor of $P(x)$
  - $P(a) = 0$
  - a is a solution, root or zero of $P(x)$

Rational Root Theorem

All possible rational roots (solutions, zeros) of $P(x)$ are:

$$\frac{p}{q}$$

p ← integer factors of constant

q ← integer factors of leading coefficient

Find all possible rational roots/zeros:

1) $P(x) = 1x^3 + x^2 - 3x + 6$

$$\frac{p}{q} = \frac{\pm 1 \ \pm 2 \ \pm 3 \ \pm 6}{\pm 1} = \pm 1 \ \pm 2 \ \pm 3 \ \pm 6$$

2) $P(x) = 3x^3 - x^2 - 15x + 5$

$$\frac{p}{q} = \frac{\pm 1 \ \pm 5}{\pm 1 \ \pm 3} = \pm 1 \ \pm \frac{1}{3} \ \pm 5 \ \pm \frac{5}{3}$$

Find the roots of $x^3 - 5x^2 + 7x - 35 = 0$

1) Determine possible rational roots:

$$\frac{p}{q} = \frac{\pm 1 \ \pm 5 \ \pm 7 \ \pm 35}{\pm 1} = \pm 1 \ \pm 5 \ \pm 7 \ \pm 35$$

2) Test possibilities (synthetically)

1	1	-5	7	-35
		1	-4	3
	1	-4	3	X

5	1	-5	7	-35
		5	0	35
	1	0	7	☺

$$(x-5)(x^2+7) = P(x)$$

3) Solve Quadratic Or Repeat 1-2

$$x^2 + 7 = 0$$

$$\sqrt{x^2} = \sqrt{-7}$$

$$x = \pm i\sqrt{7}$$

Roots/Zeros/Solutions = $\{5, i\sqrt{7}, -i\sqrt{7}\}$

Find all zeros: $y = 2x^3 + 14x^2 + 13x + 6$

1) Possible rational zeros

$$\frac{p}{q} = \frac{\pm 1 \;\pm 2 \;\pm 3 \;\pm 6}{\pm 1 \;\pm 2} = \pm 1 \;\pm \tfrac{1}{2} \;\pm 2 \;\pm 3 \;\pm \tfrac{3}{2} \;\pm 6$$

2) Test Possibilities (Hint: Desmos.com)

-6	2	14	13	6
		-12	-12	-6
	2	2	1	☺

Thanks Desmos

↓

$2x^2 + 2x + 1$

3) Solve Quadratic or Repeat Steps 1-2

$$2x^2 + 2x + 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)} = \frac{-2 \pm \sqrt{-4}}{4} = \frac{-1 \pm i}{2}$$

$$\text{zeros} = \left\{ -6, \frac{-1 \pm i}{2} \right\}$$

Irrational Root Theorem

If $(a+\sqrt{b})$ is a root of $P(x)$ then its conjugate $(a-\sqrt{b})$ is also a root

Imaginary Root Theorem

If $(a+bi)$ is a root of $P(x)$ then its conjugate $(a-bi)$ is also a root

Irrational/Imaginary Roots Come In Pairs!

Solutions/Roots/Zeros of

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

n Complex Roots } Fundamental Theorem of Algebra (FTA)

- Real
  - Rational: $\frac{p}{q}$
  - Irrational: $a+\sqrt{b}$, $a-\sqrt{b}$
- Imaginary: $a+bi$, $a-bi$

Review Write A Polynomial With Zeros 1, -1 and 4

$$P(x) = (x-1)[(x+1)(x-4)]$$ Write as factors

$$= (x-1)(x^2-3x-4)$$ FOIL any pair

I) $P(x) = x^3 - 3x^2 - 4x - x^2 + 3x + 4$

$P(x) = x^3 - 4x^2 - x + 4$

II)

	x^2	$-3x$	-4
x	x^3	$-3x^2$	$-4x$
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$P(x) = x^3 - 4x^2 - x + 4$

III)

$$\begin{array}{r} x^2 - 3x - 4 \\ x - 1 \\ \hline -x^2 + 3x + 4 \\ x^3 - 3x^2 - 4x \\ \hline x^3 - 4x^2 - x + 4 \end{array}$$

Write a polynomial equation with the following roots: $-3,\ 3+i,\ 3-i$

$$P(x) = (x+3)\left(x-(3+i)\right)\left(x-(3-i)\right)$$

$$P(x) = (x+3)\left[\left((x-3)-i\right)\left((x-3)+i\right)\right] \quad \text{FOIL}$$

$$P(x) = (x+3)(x^2-6x+9-i^2)$$

$$P(x) = (x+3)(x^2-6x+10)$$

$$P(x) = x^3-3x^2-8x+30$$

$P(x) = x^4 - 3x^3 + x^2 - x + 3$

Complex Roots = 4

Possible # Real Roots = 4, 2 or 0

Possible # Real Positive Roots = 4, 2 or 0

Possible # Negative Real Roots = 0

Possible Rational Roots = ± 1 ± 3