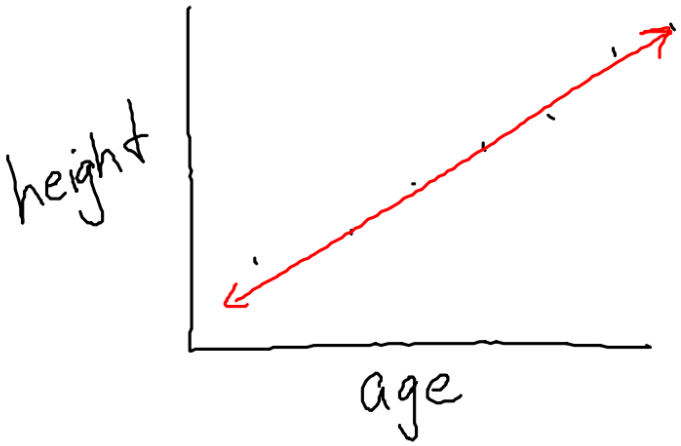
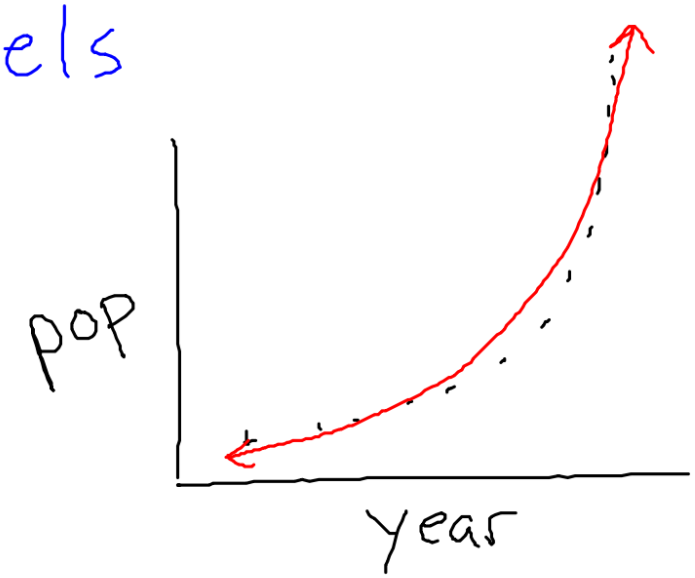
Sec 8-1

Regression Models



Linear

$y = ax + b$



Exponential

$y = ab^x$

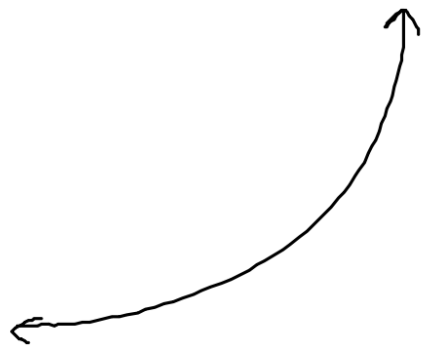
$(a \neq 0,\ b > 0,\ b \neq 1)$

Exponential Growth ($b > 1$)

Ex $y = 3^x$

$y = 4(2)^x$

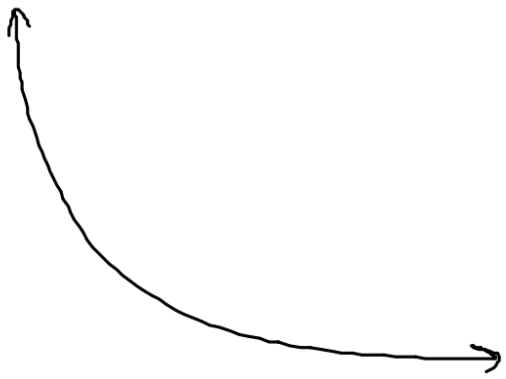
$y = 4 \cdot 2^x$



Exponential Decay ($0 < b < 1$)

Ex $y = \left(\frac{1}{2}\right)^x$

$y = 3(.9)^x$



Finding Growth/Decay Factors

$$b = 1 + r$$

↑ Rate of increase/decrease

Ex If $y = 2(1.6)^x$ find the annual percent increase

$b = 1 + r$

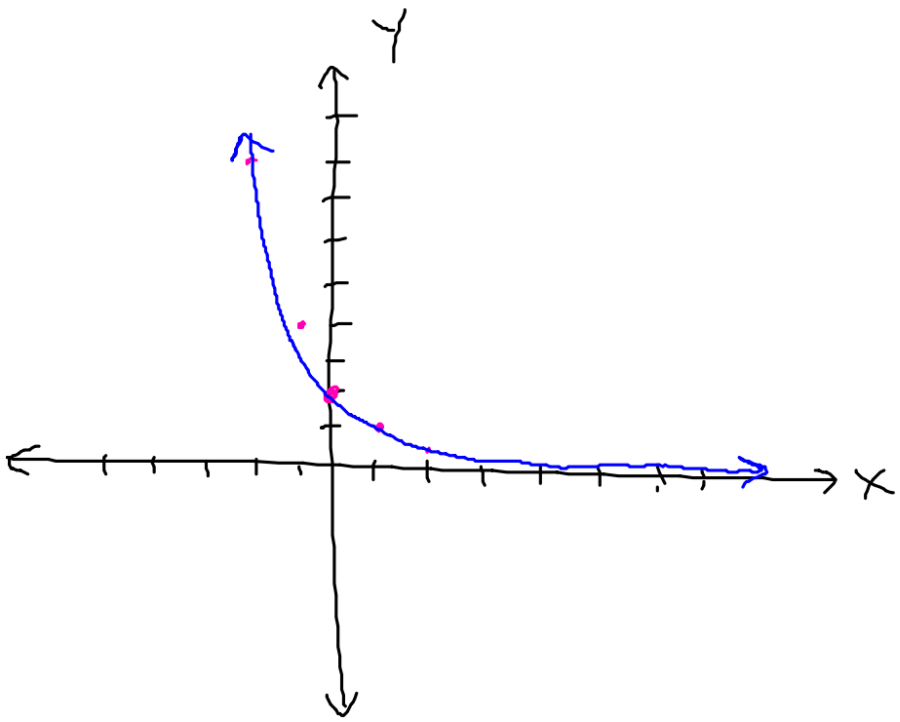
$1.6 = 1 + r$

$.6 = r = 60\%$

Graphing

$$y = 2\left(\frac{1}{2}\right)^x$$

X	Y
0	2
1	1
2	$\frac{1}{2}$
-1	4
-2	8



Writing Exponential Function (2,4) (3,16)

$y = ab^x$

$4 = ab^2$

$\frac{4}{b^2} = a$

$y = ab^x$

$16 = \left(\frac{4}{b^2}\right)b^3$

$16 = \frac{4b^3}{b^2}$

$16 = 4b$

$4 = b$

$a = \frac{4}{b^2}$

$a = \frac{4}{4^2}$

$a = \frac{4}{16} = \frac{1}{4}$

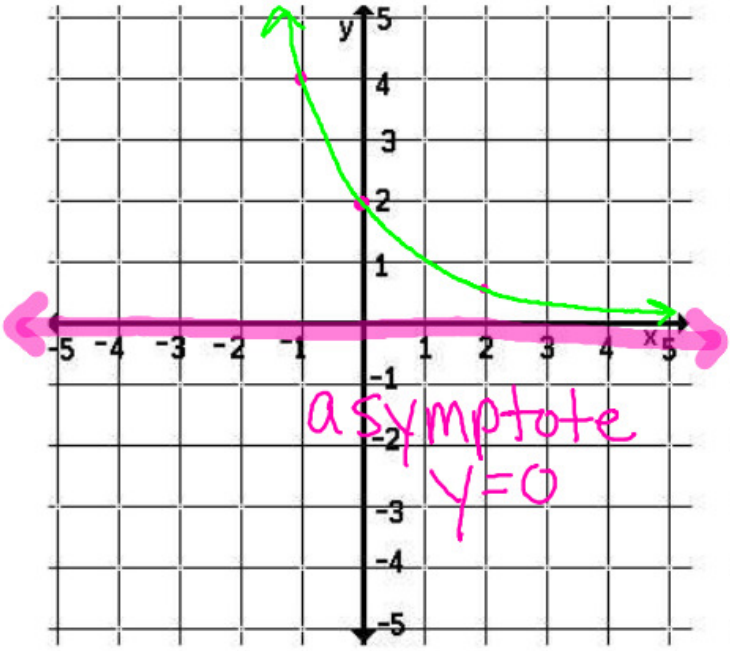
$\downarrow$

$y = \frac{1}{4}(4)^x$

Sec 8-2

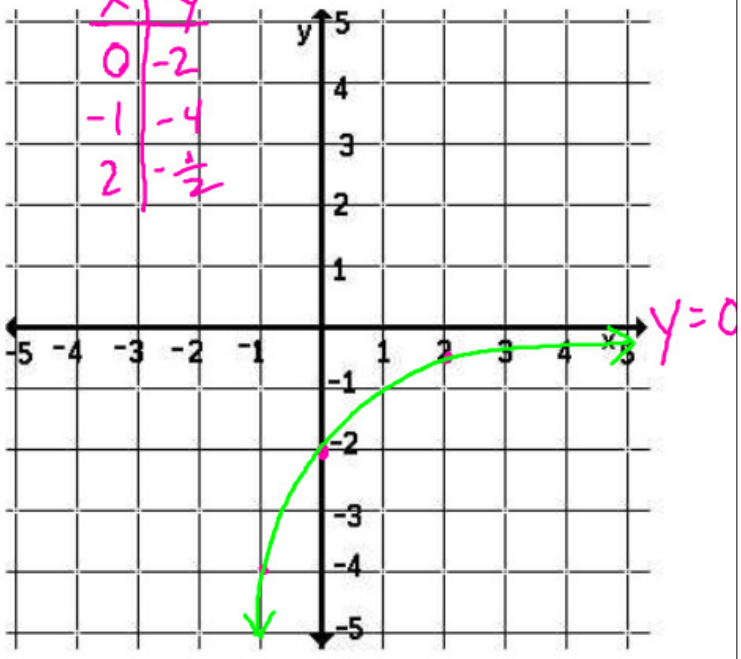
GRAPHING EXPONENTIAL EQUATIONS

1. $y = 2\left(\frac{1}{2}\right)^x$

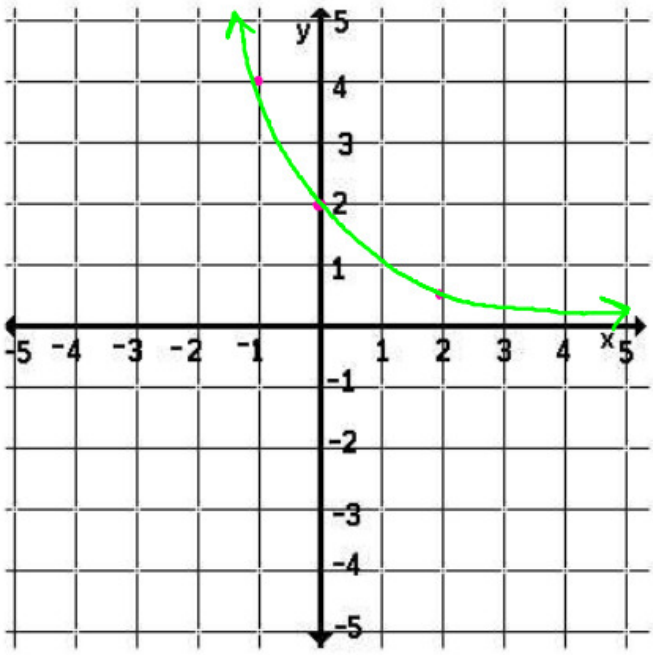


$y = -2\left(\frac{1}{2}\right)^x$

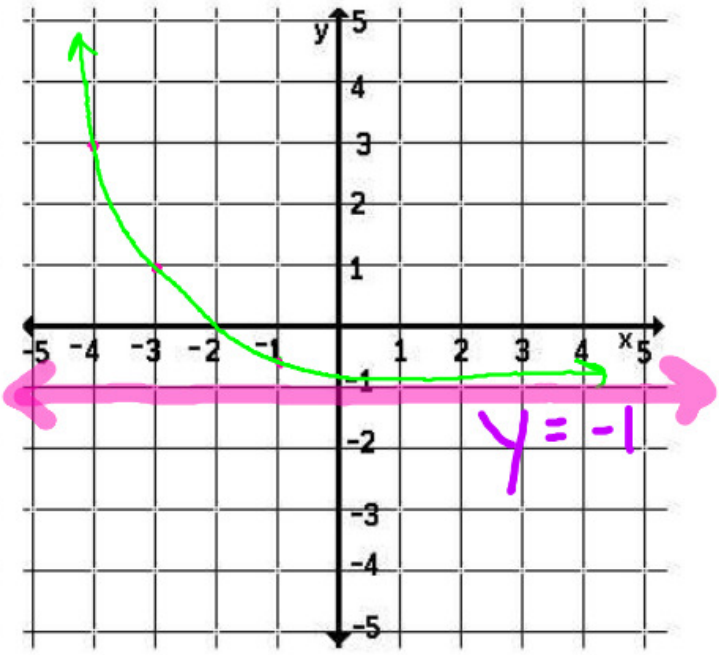
x	y
0	-2
-1	-4
2	$-\frac{1}{2}$



2. $y = 2\left(\frac{1}{2}\right)^x$



$y = 2\left(\frac{1}{2}\right)^{x+3} - 1$



In General

$$y = a(b)^{x-h} + k$$

Just Like...

$$y = a(x-h)^2 + k$$

$$y = a|x-h| + k$$

$$y = a\sqrt{x-h} + k$$

Definitions of Famous Numbers

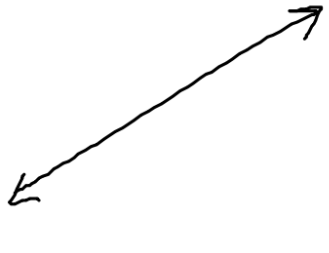
Pi $\pi \approx 3.14$

Phi $\Phi \approx 1.618$

$e \approx 2.718$

Simple Interest

$$I = prt$$



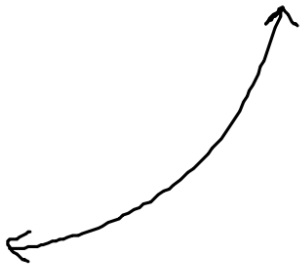
Continuously Compounded Interest

$$A = pe^{rt}$$

Rate → r ← Time (years)

A: Amount

p: Principal



Ex Invest $1300 at annual interest rate of 4.3% compounded continuously for 3 years; how much will you have?

$$A = Pe^{rt}$$

$$A = (1300)(2.718)^{.043(3)}$$

$$A = (1300)(2.718)^{.129}$$

$$A = \$1478.98$$

Review
(Sec 2-1)

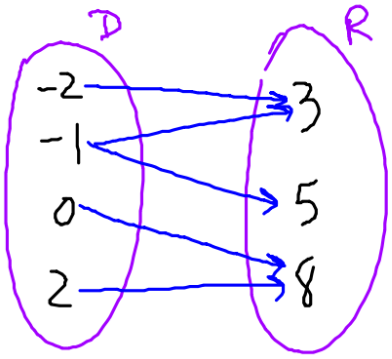
Relation

Domain

Range

Set of (x, y) pairs of numbers

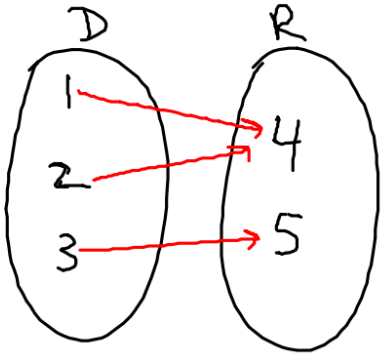
Ex $\{(2,8)\ (-1,5)\ (0,8)\ (-1,3)\ (-2,3)\}$



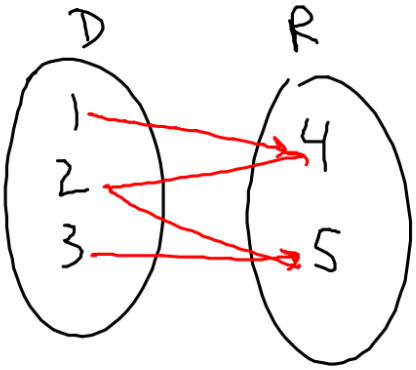
Mapping Diagram

Function

Relation where each element in domain is paired with exactly one element in range



Function



Not Function

Function Notation

$$f(x) = 2x - 3$$

"f of x"

Ex Find $f(4) \rightarrow 2(4) - 3 = 5$

Sec 7-7

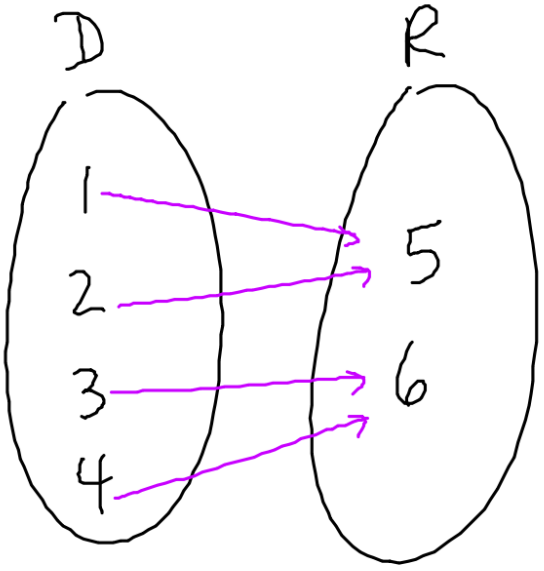
Ex $g(x) = 2x$ $h(x) = x^2 + 4$

Find $h(-2) = (-2)^2 + 4 = 8$

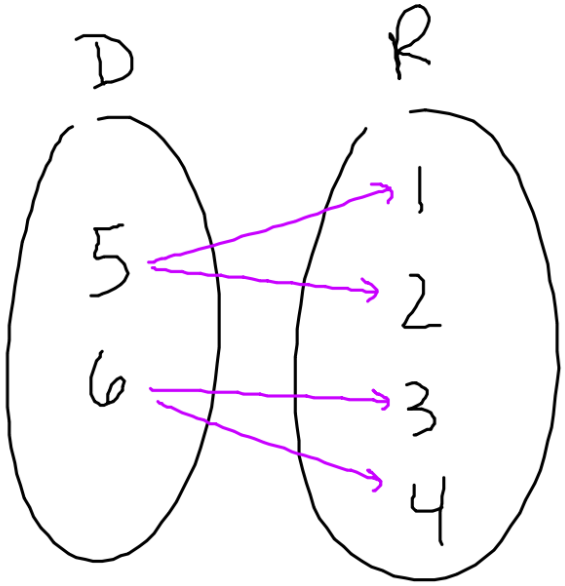
$g(8) = 2(8) = 16$

$g(h(-2)) = (g \circ h)(-2)$

Inverse Relations



Relation (Function)



Inverse (Not Function)

Finding An Inverse $f^{-1}(x)$

Ex $f(x) = 3x - 4$

$y = 3x - 4$ > Function

↓ Inverse

$x = 3y - 4$

$3y - 4 = x$

$y = \frac{1}{3}x + \frac{4}{3}$

$f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$ > Function

Ex $f(x) = x^2 - 2$

$y = x^2 - 2$ > Function 

↓ Inverse

$x = y^2 - 2$

$y^2 - 2 = x$

$y^2 = x + 2$

$y = \pm\sqrt{x+2}$

$f^{-1}(x) = \pm\sqrt{x+2}$ > Not Function 

Properties

1) The inverse of a function is sometimes a function

2) If $f(x)$ and $f^{-1}(x)$ are inverses then $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$

Ex $f(x) = 2x$ and $f^{-1}(x) = \frac{1}{2}x$

$(f^{-1} \circ f)(6) = ?$

$f(6) = 2(6) = 12$

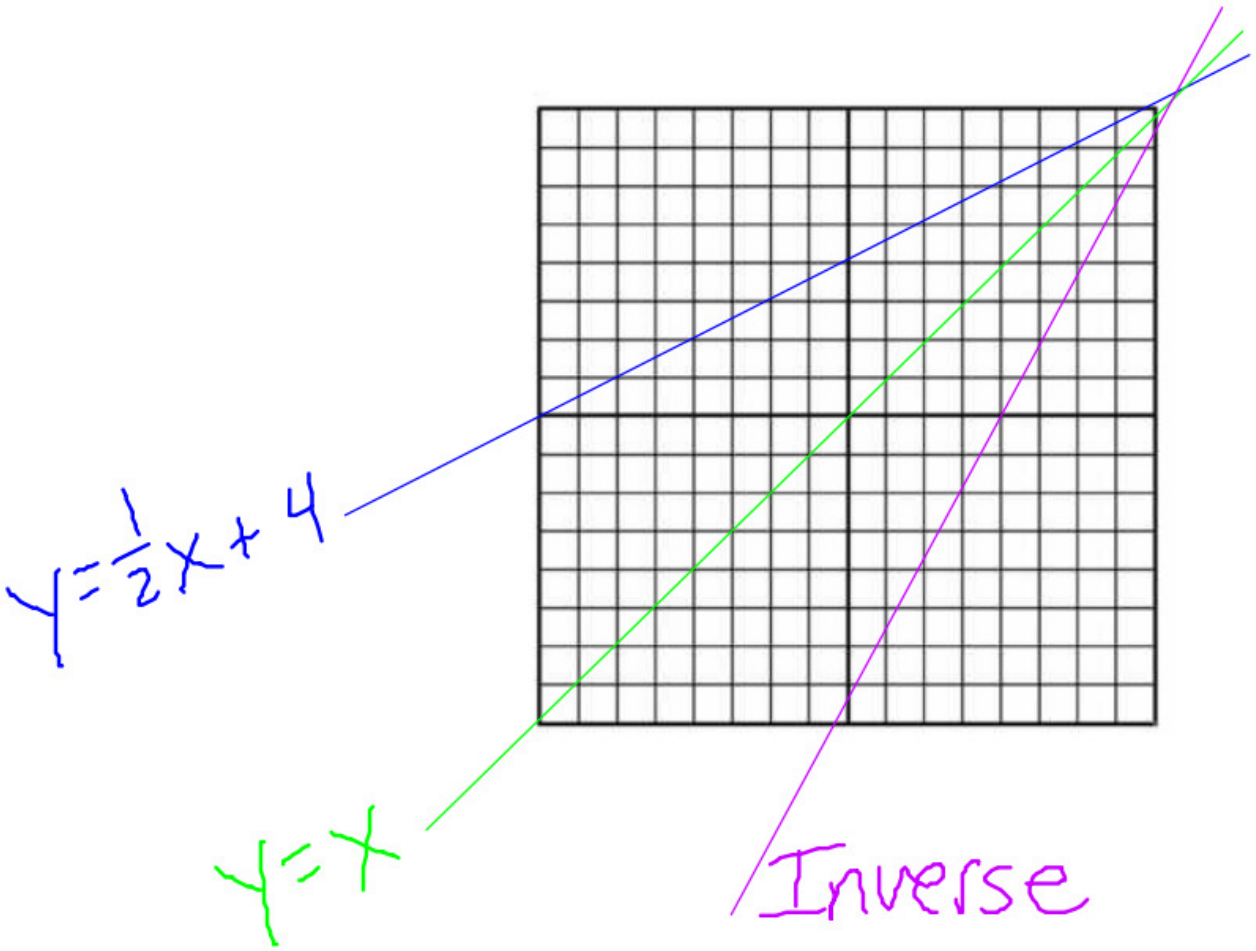
$f^{-1}(12) = \frac{1}{2}(12) = 6$

Graphing Inverses

1) Use $y = x$ Reflections

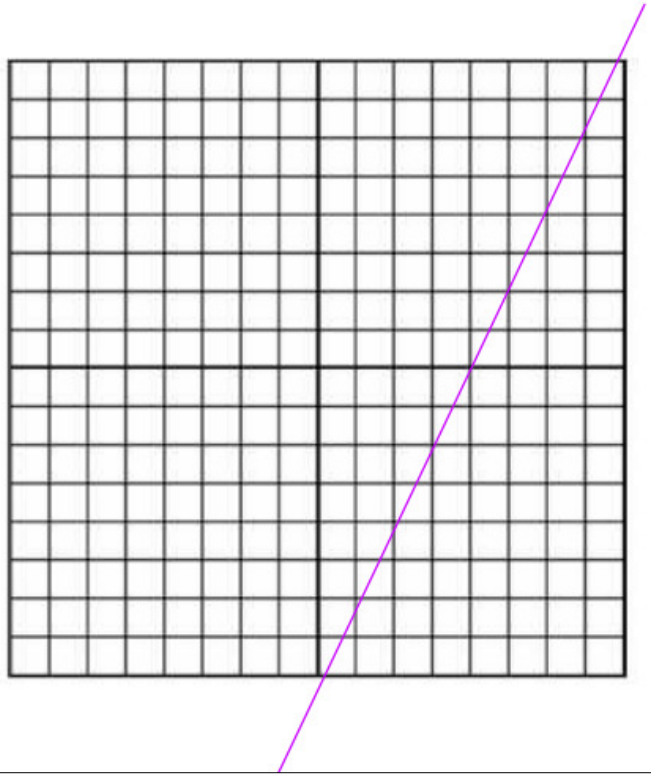
2) Use Inverse Solutions

Ex Graph the inverse of $f(x) = \frac{1}{2}x + 4$



X	Y
0	4
2	5
-2	3

X	Y
4	0
5	2
3	-2



Sec 8-3

Exponential	Inverse	Logarithmic Form
$y = b^x$	$x = b^y$	$y = \log_b x$
$y = 10^x$	$x = 10^y$	$y = \log x$

base 10 implied

Write In Log Form

Ex $729 = 3^6$

$\boxed{6} = \log_{\boxed{3}} \boxed{729}$

"6 equals log 729 base 3"

Ex $32 = 2^5$

$5 = \log_2 32$

Evaluate

Ex $\log_9 27 = x$

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = 3/2$

$5^{2x} = 3^3$

Ex $\log 100 = x$

$10^x = 100$

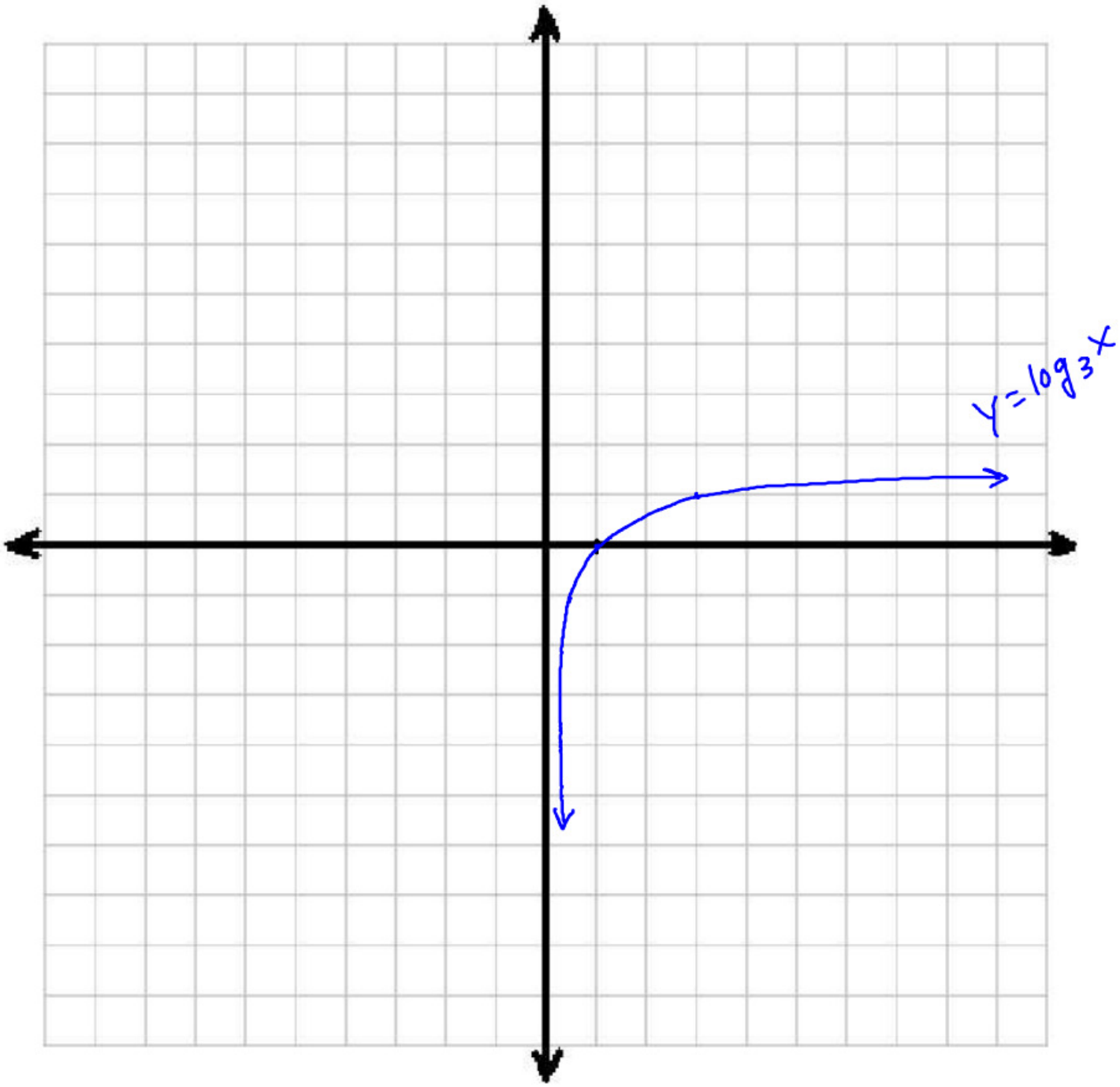
$10^x = 10^2$

$x = 2$

Graphing Logs

$$y = \log_3 x \longrightarrow x = 3^y$$

X	Y
1	0
3	1
$\frac{1}{3}$	-1


$y = \log_3 x$

Sec 8-4

$$\left[x^2 \cdot x^3 = x^{2+3} = x^5 \right]$$

1) $\log_b MN = \log_b M + \log_b N$

$$\log_2 8 = \log_2 4 + \log_2 2$$

$$\log 2 + \log 3 = \log 6$$

$$\left[\frac{x^5}{x^2} = x^{5-2} = x^3\right]$$

2) $\log_b \frac{M}{N} = \log_b M - \log_b N$

$$\log_3 20 - \log_3 4 = \log_3 \frac{20}{4} = \log_3 5$$

3) $\log_b M^x = x \log_b M$

$\log 10^2 = 2(\log 10) = 2(1) = 2$ ✓

$\frac{1}{3} \log_5 8 = \log_5 8^{\frac{1}{3}} = \log_5 2$

Evaluate

$$2\log_8 4 - \frac{1}{3}\log_8 8 = x$$

$$\log_8 16 - \log_8 2 = x$$

$$\log_8 8 = x$$

$$8^x = 8^1$$

$$x = 1$$

Sec 8-5

Evaluate (Using A Calculator)

$\log 8 \approx .9031 \quad [10^{.9031} = 8]$

$\log 50 \approx 1.6990 \quad [10^{1.6990} = 50]$

$\log_2 7 = ?$

Change of Base Formula

$$\log_b M = \frac{\log M}{\log b}$$

Ex $\log_2 7 = \frac{\log 7}{\log 2} \approx 2.8074$

Convert $\log_2 7$ to base 8.

$$\log_2 7 = \log_8 x$$

$$2.8074 = \log_8 x$$

$$\boxed{8}^{\boxed{2.8074}} = \boxed{x}$$

$$343 = x$$

Solve $6^x = 21$

$$\log 6^x = \log 21$$

$$x \log 6 = \log 21$$

$$x = \frac{\log 21}{\log 6} = 1.6992$$

Solve $4^x = 32$

$(2^2)^x = 2^5$

$2^{2x} = 2^5$

$2x = 5$

$x = 5/2$

$\log 4^x = \log 32$

$x \log 4 = \log 32$

$x = \frac{\log 32}{\log 4} = 2.5$

Solve $7^{5x} = 3000$

$$\log 7^{5x} = \log 3000$$

$$\frac{5x \log 7}{5 \log 7} = \frac{\log 3000}{5(\log 7)}$$

$$X = .8229$$

Solve $\log(7-2x) = -1$

$$7 - 2x = 10^{-1}$$

$$7 - 2x = .10$$

$$-2x = -6.9$$

$$x = 3.45$$

Solve $\log 6 - \log 3x = -2$

$$\log \frac{6}{3x} = -2$$

$$10^{-2} = \frac{6}{3x}$$

$$\frac{1}{100} = \frac{6}{3x}$$

$$3x = 600$$

$$x = 200$$

Sec 8-6

Common Log

A logarithm with base 10

$$\log_{10} x = \log x$$

10 implied

Natural Log

A logarithm with base e.

$e \approx 2.7183$

$$\log_e x = \ln x$$

Write as a single log:

$$5\ln 2 - \ln 4$$

$$= \ln 2^5 - \ln 4$$

$$= \ln 32 - \ln 4$$

$$= \ln \frac{32}{4}$$

$$= \ln 8$$

Solve $\ln(3x-9) = 4$

$$e^{4} = 3x-9$$

$$e^4 + 9 = 3x$$

$$\frac{e^4+9}{3} = x$$

$$21.1994 = x$$

Solve $e^{x+1} = 30$

$$\ln e^{x+1} = \ln 30$$

$$x+1\,(\ln e) = \ln 30$$

($\ln e = 1$)

$$x+1 = \ln 30$$

$$x = \ln 30 - 1$$

$$x = 2.4012$$

How many years would it take to double your money at 8% compounded continuously?

$A = pe^{rt}$

$2p = pe^{rt}$

$2 = e^{rt}$

$2 = e^{.08t}$

$\ln 2 = \ln e^{.08t}$

$\ln 2 = .08t(\ln e)$ ($\ln e = 1$)

$\frac{\ln 2}{.08} = t \approx 8.66$ yrs