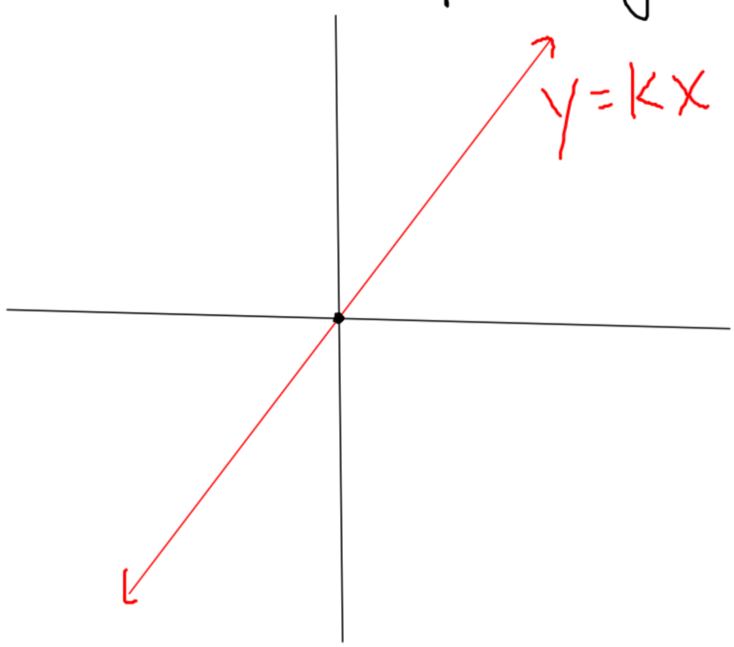
Secs 2-3 and 9-1

Statement	Equation
y varies directly as x	$y = kx$ (constant of variation)
y varies directly as x^2	$y = kx^2$
y varies jointly with x and z	$y = kxz$

Note

A direct variation ($y = kx$) is a linear function passing through $(0,0)$



Statement	Equation
y varies inversely as x	$y = \frac{k}{x}$
y varies directly as x and y and inversely as z	$y = \frac{kxy}{z}$

Ex y varies directly as x. If $y = 4$ when $x = -2$, find x when $y = 16$.

$y = kx$

$4 = k(-2)$

$-2 = k$

$y = -2x$

$16 = -2x$

$\boxed{-8 = x}$

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\frac{-2}{4} = \frac{x_2}{16}$$

$4x_2 = -32$

$\boxed{x_2 = -8}$

Ex X and y vary inversely. If $x=5$ when $y=1.6$, find y when $x=16$

$Y = \frac{K}{X}$

$\frac{1.6}{1} = \frac{K}{5}$

$K = 8$

$Y = \frac{8}{X}$

$Y = \frac{8}{16}$

$\boxed{Y = \frac{1}{2}}$

$X_1 Y_1 = X_2 Y_2$

$(5)(1.6) = (16)(Y_2)$

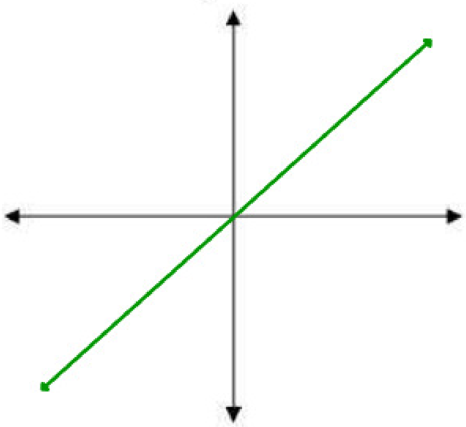
$8 = 16 Y_2$

$\frac{8}{16} = \boxed{Y_2 = \frac{1}{2}}$

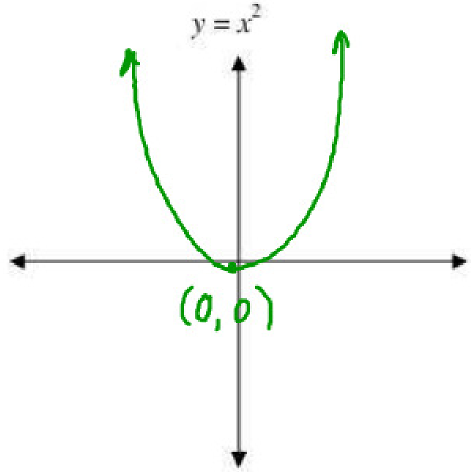
Sec 9.2

GRAPHS

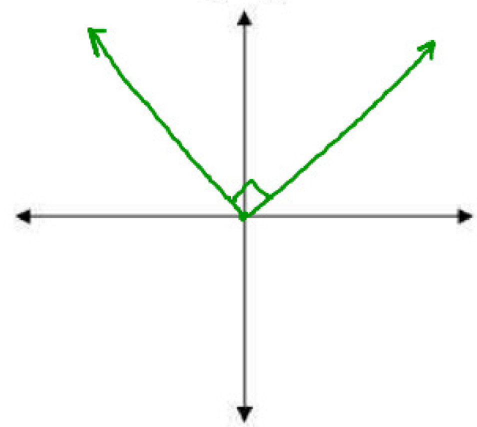
$y = x$



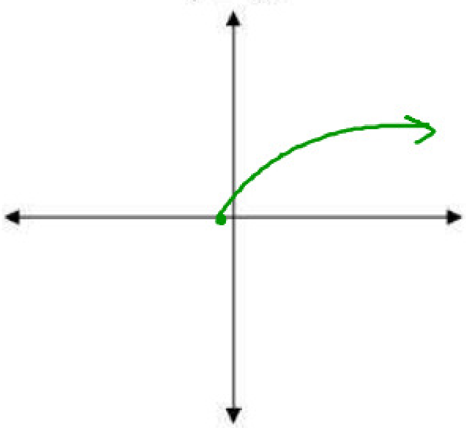
$y = x^2$



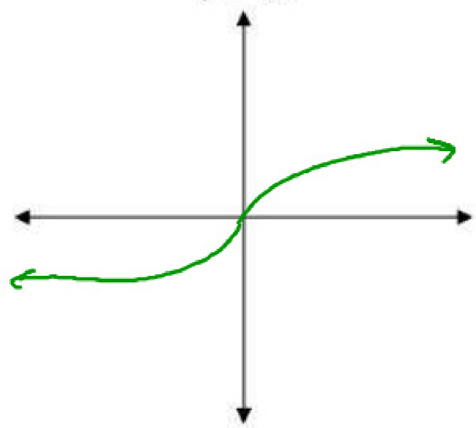
$y = |x|$



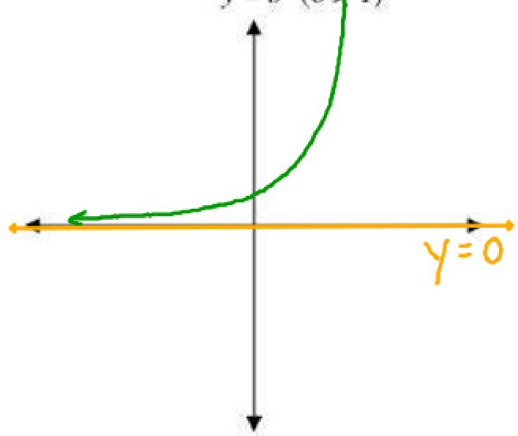
$y = \sqrt{x}$



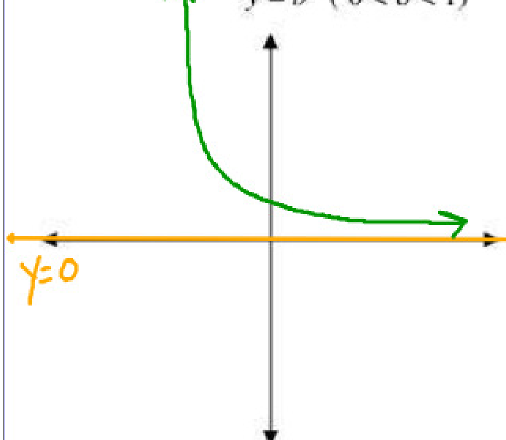
$y = \sqrt[3]{x}$



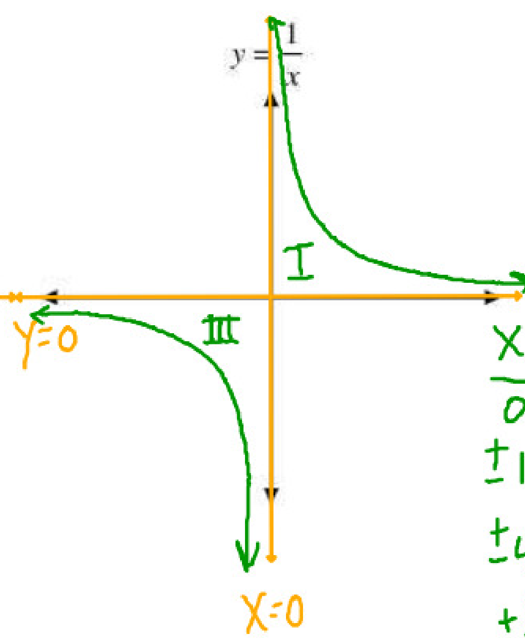
$y = b^x$ (b > 1)



$y = b^x$ (0 < b < 1)

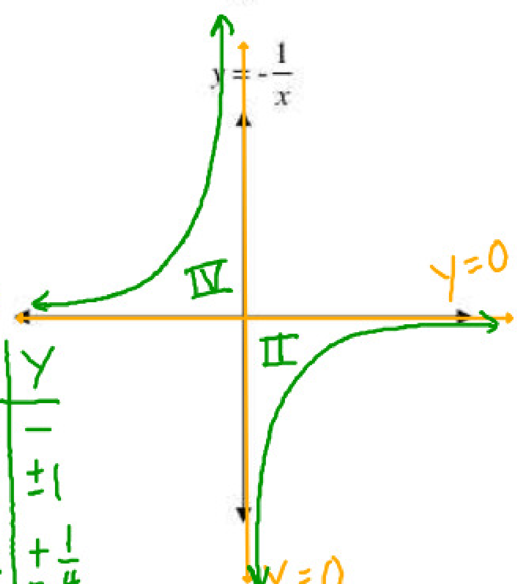


$y = \frac{1}{x}$



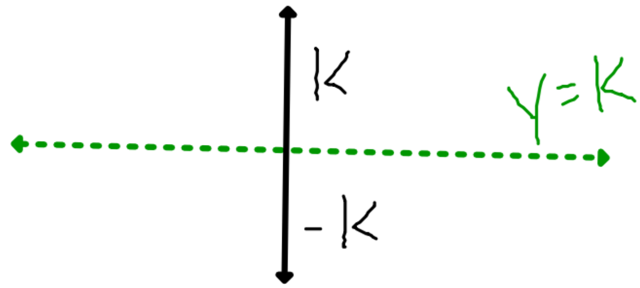
X	Y
0	–
± 1	± 1
± 4	$\pm \frac{1}{4}$
$\pm \frac{1}{4}$	± 4

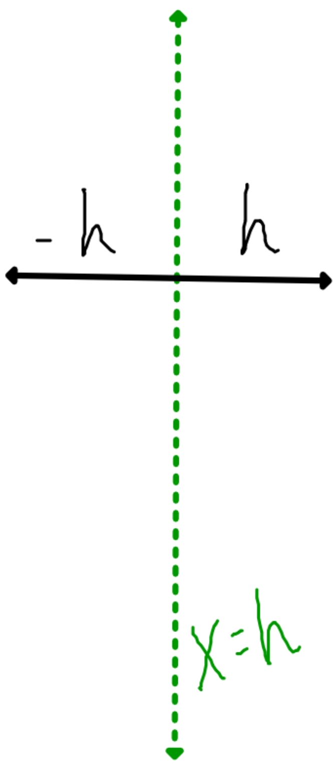
$y = -\frac{1}{x}$



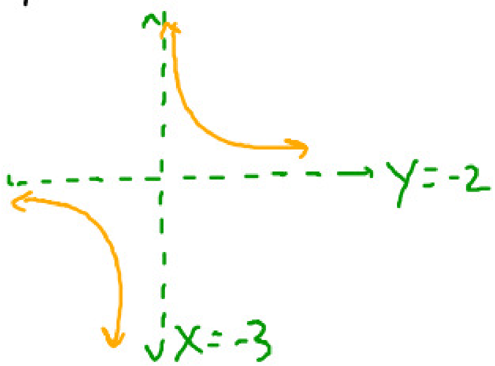
Translations of Inverse Variations

$$y = \frac{a}{x-h} + K$$



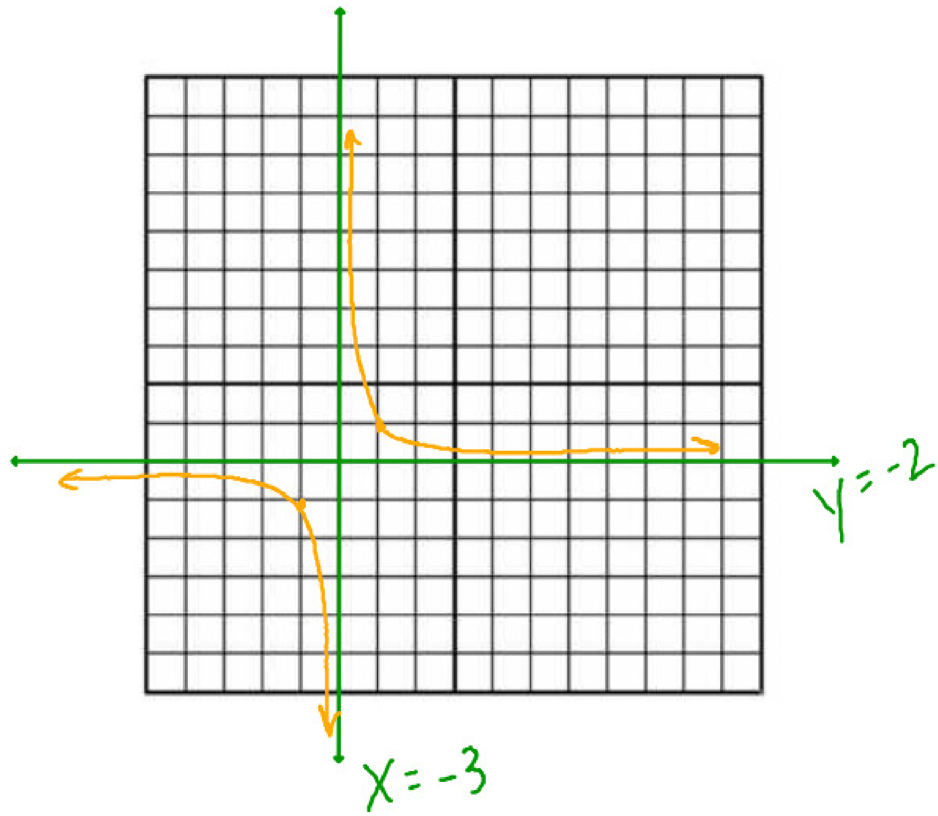


$$y = \frac{1}{x+3} - 2$$

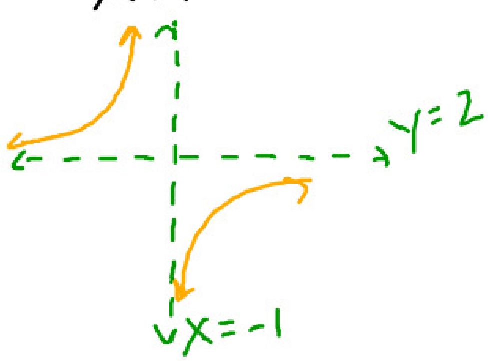


Denom = ±1

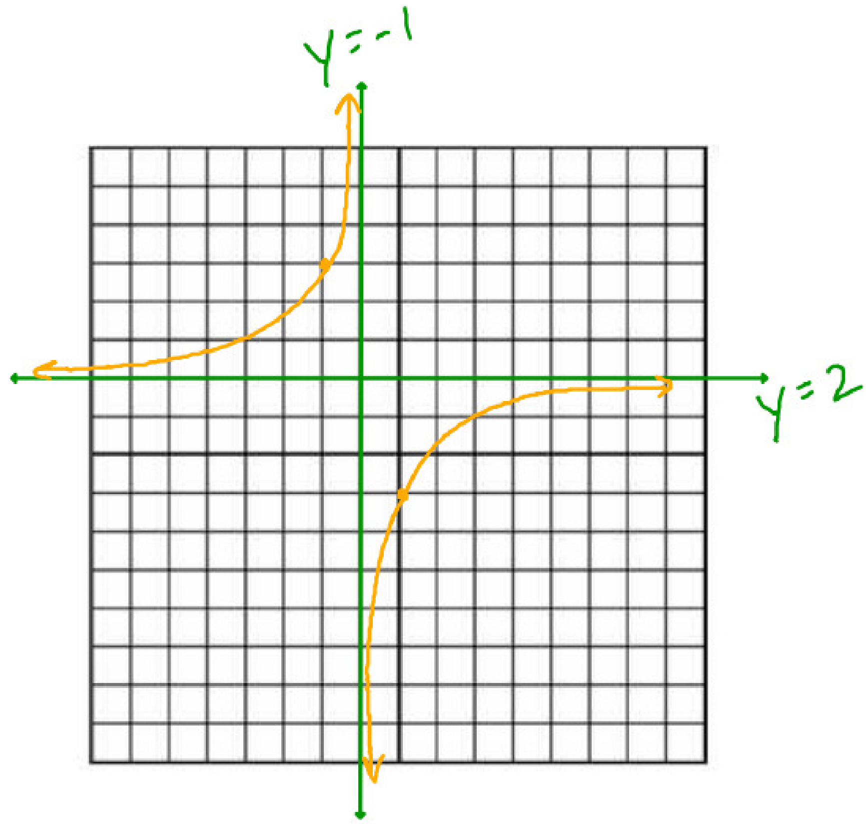
X	Y
-2	-1
-4	-3



$$y = \frac{-3}{x+1} + 2$$



X	Y
0	-1
-2	5



Sec 9-3

GRAPHING RATIONAL FUNCTIONS

$$f(x) = \frac{P(x)}{Q(x)}$$

1. **a.** If possible, factor the numerator $P(x)$ and denominator $Q(x)$

b. Determine points of discontinuity (if any); determine vertical asymptotes (if any) using each real zero of the denominator $Q(x)$

c. Graph and label

2. **a.** Determine horizontal asymptote (if any) using degrees of the numerator $P(x)$ and denominator $Q(x)$:

I.	Degree of $P(x)$ < Degree of $Q(x)$	$y = 0$
II.	Degree of $P(x)$ > Degree of $Q(x)$	None
III.	Degree of $P(x)$ = Degree of $Q(x)$	$y = \frac{a}{b}$

b. Graph and label

3. **a.** Choosing x-values near the vertical asymptotes, calculate y-values

b. Finish graphing

$$y = \frac{x+3}{x^2-6x+5} = \frac{x+3}{(x-1)(x-5)}$$

1. ~~Points of discontinuity~~/vertical asymptotes:

$x - 1 = 0 \qquad x - 5 = 0$

$x = 1 \qquad x = 5$

2. Horizontal asymptote:

Num < Denom

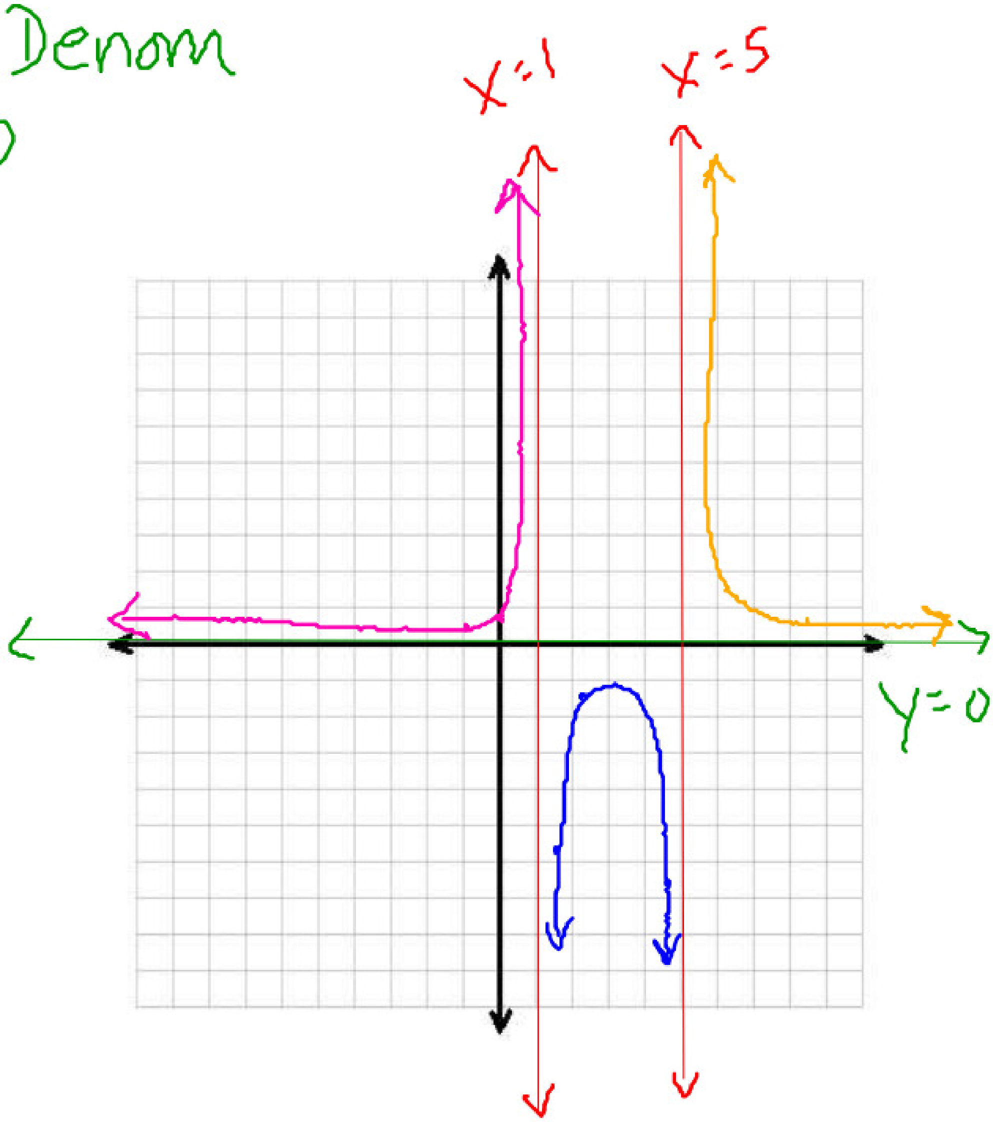
$y = 0$

$$y = \frac{x+3}{x^2-6x+5}$$

X	Y
0	.6
-1	.2
$\frac{1}{2}$	4

X	Y
5.1	19.8
6	1.8
8	.5

X	Y
1.2	-5.5
2	-1.7
4.6	-5.3



$$y = \frac{4x-1}{x+2}$$

1. ~~Points of discontinuity~~/vertical asymptotes:

$x + 2 = 0$

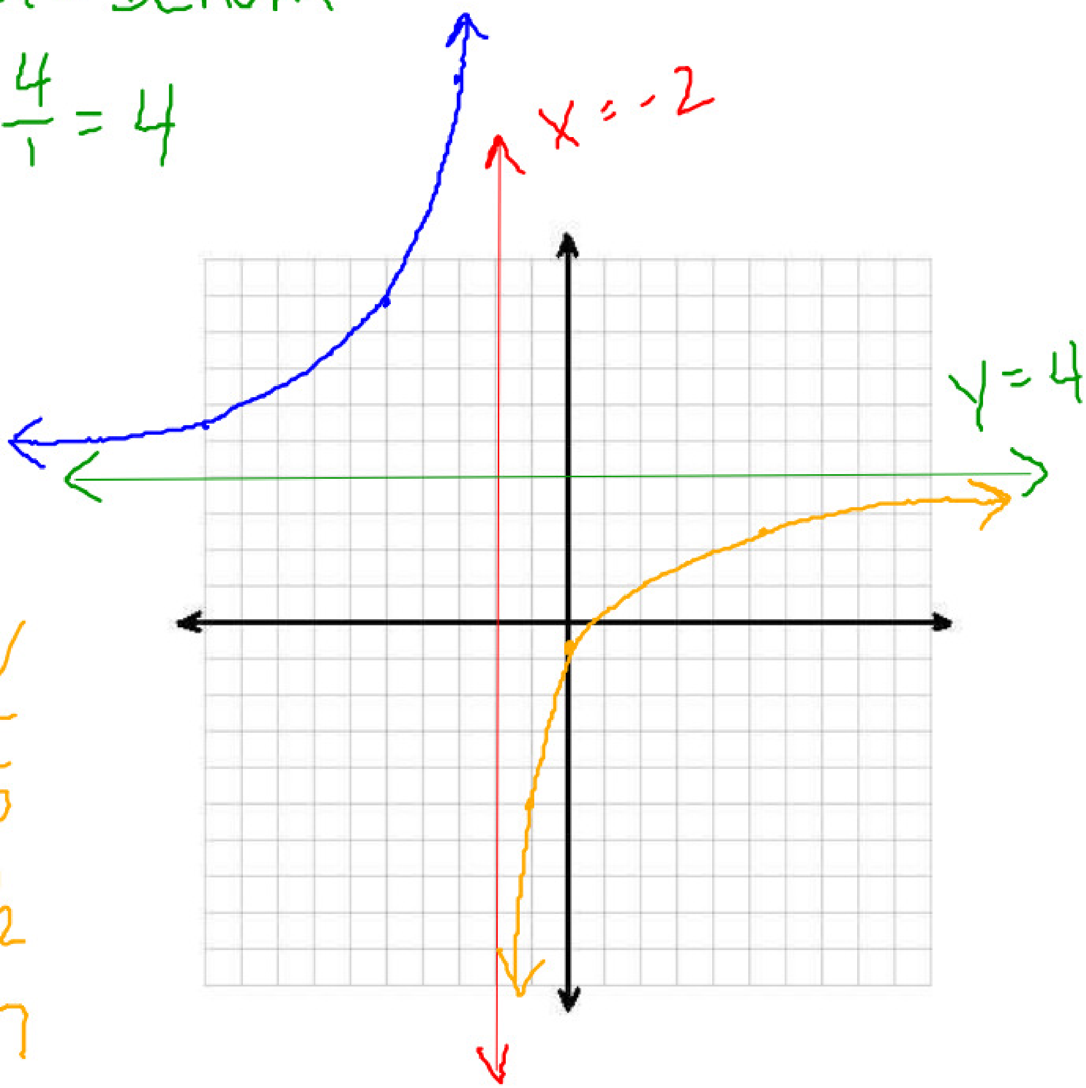
$x = -2$

2. Horizontal asymptote:

Num = Denom

$y = \frac{4}{1} = 4$

$$y = \frac{4x-1}{x+2}$$



x	y
-3	13
-5	7
-10	$5\frac{1}{8}$

x	y
-1	-5
0	$-\frac{1}{2}$
5	2.7

$y = \frac{x^2 + 2x}{x + 2}$ $= \frac{x\cancel{(x+2)}}{\cancel{x+2}}$

1. Points of discontinuity/vertical asymptotes:

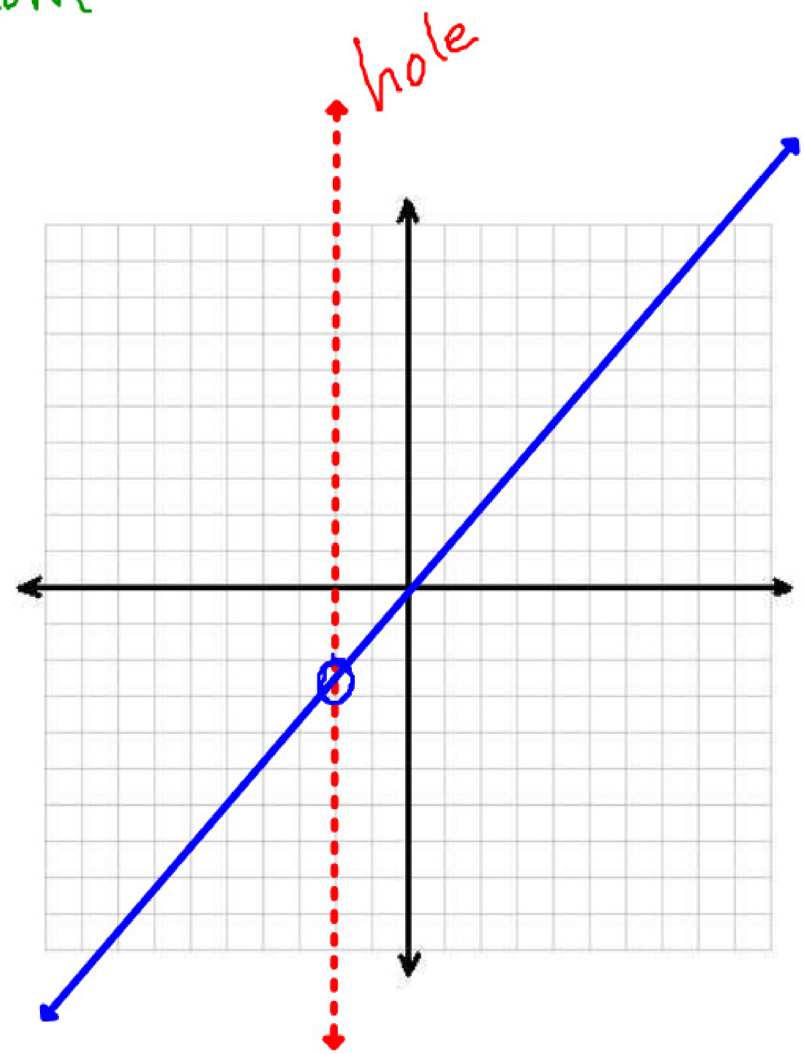
At $x = -2$ None

2. Horizontal asymptote:

Num > Denom

None

$y = x$



$$y = \frac{x-2}{x^2+x-6} = \frac{\cancel{x-2}^{\,1}}{(\cancel{x-2})(x+3)}$$

1. Points of discontinuity/vertical asymptotes:

Hole at $x = 2$

$x + 3 = 0$

$x = -3$

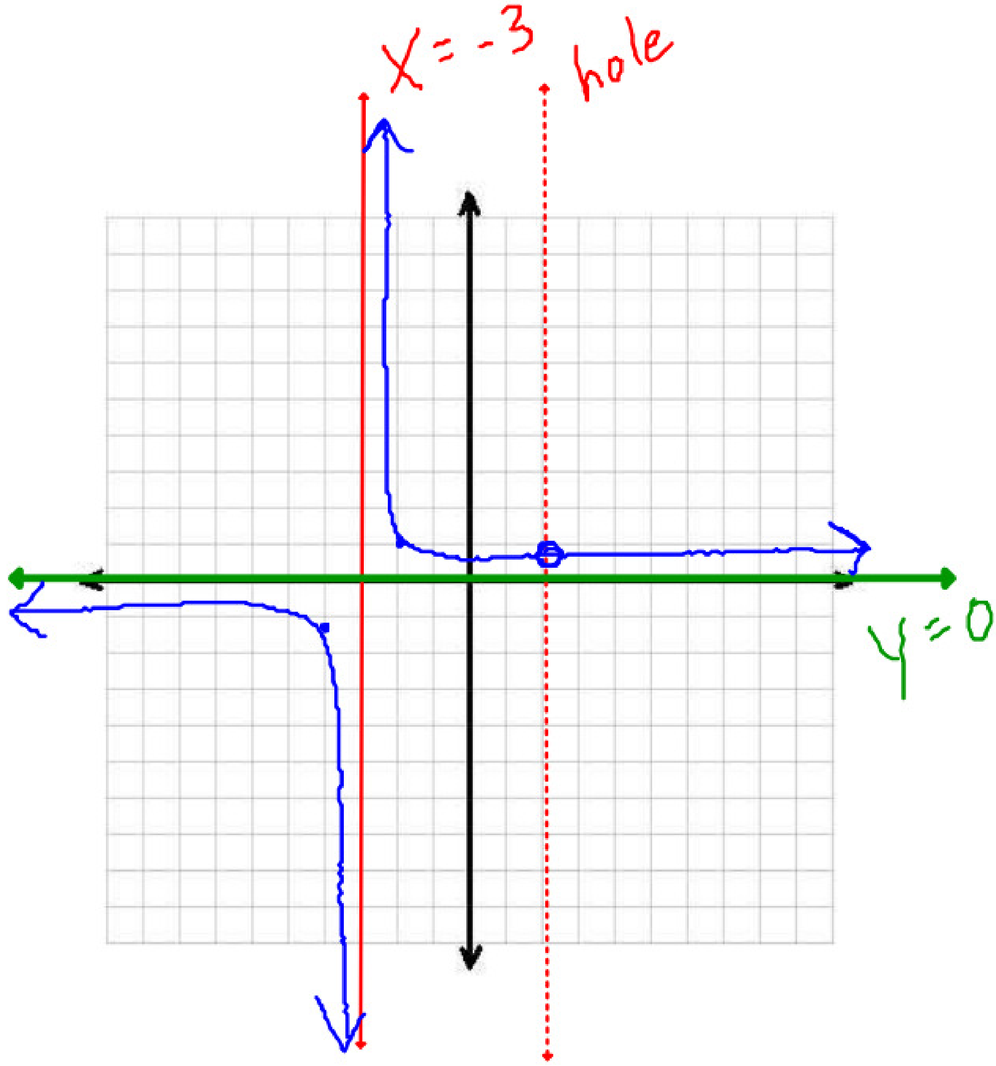
2. Horizontal asymptote:

Num < Denom

$y = 0$

$$y = \frac{1}{x+3}$$

x	y
-2	1
-4	-1



$$y=\frac{\overset{1}{\cancel{x-2}}}{x^2+x-6} \rightarrow y=\frac{1}{x+3}$$

$(x+3)(\cancel{x-2})$

X	Y
-2	1
-4	-1

1. Vertical asymptotes/holes of discontinuity:

$X=-3$ At $X=2$

2. Horizontal asymptote:

Num < Denom

$Y=0$

$$y=\frac{1}{x+3}$$

X	Y
-2	1
-4	-1