Algebra II

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Algebra II:

Standard 1 — Relations and Functions

Students recognize and graph polynomial, rational, and algebraic functions. They understand the concept of functional notation and use it to combine functions by composition. They solve equations and inequalities by examining their graphs and interpret situations as functions in graphs, formulas, and words.

Standard 2 — Linear and Absolute Value Equations and Inequalities

Students graph linear equations and inequalities involving absolute value. They use a variety of methods to solve systems of up to three linear equations in up to three variables, and they model data with linear equations and make predictions from the results.

Standard 3 — Quadratic Equations and Functions

Students extend the number system by defining complex numbers, relating them to the real numbers, and using them to solve quadratic equations. They draw graphs of quadratic functions and apply transformations to the functions. They find and interpret zeros and maximum and minimum values, and solve word problems. They also solve equations containing radicals and solve pairs of equations.

Standard 4 — Conic Sections

Students write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one.

Standard 5 — Polynomials

Students understand and use the binomial theorem for positive integer powers. They learn techniques for factoring polynomials in order to solve equations and related word problems. They find approximate solutions of equations using graphing technology and write equations with given solutions. They understand the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial.

Standard 6 — Algebraic Fractions

Students understand and use the concepts of negative and fractional exponents. They add, subtract, multiply, divide, and simplify algebraic fractions. They solve equations involving algebraic fractions and solve related word problems. They also solve problems of direct, inverse, and joint variation.

Standard 7 — Logarithmic and Exponential Functions

Students understand the concepts of logarithmic and exponential functions. They graph exponential functions and solve problems of growth and decay. They understand the inverse relationship between exponents and logarithms and use it to prove laws of logarithms and to solve equations. And they convert logarithms between bases and simplify logarithmic expressions.

Standard 8 — Sequences and Series

Students define the concepts of arithmetic and geometric sequences and series. They find specified terms of sequences and partial sums of series and use their knowledge of sequences and series to solve word problems.

Standard 9 — Counting Principles and Probability

Students understand and apply counting principles to find permutations and combinations and related probabilities.

Standard 10 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

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Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Relations and Functions

Students graph relations and functions and find zeros. They use function notation and combine functions by composition. They interpret functions in given situations.

A2.1.1 Recognize and graph various types of functions, including polynomial, rational, and algebraic functions.

Example: Draw the graphs of the functions $y = x^4 - x^2$, $y = \frac{7}{x-2}$, and $y = \sqrt{x+2}$.

A2.1.2 Use function notation. Add, subtract, multiply, and divide pairs of functions.

Example: Let $f(x) = 7x + 2$ and $g(x) = x^2$. Find the value of $f(x) \bullet g(x)$.

A2.1.3 Understand composition of functions and combine functions by composition.

Example: Let $f(x) = x^3$ and $g(x) = x - 2$. Find $f(g(x))$.

A2.1.4 Graph relations and functions with and without graphing technology.

Example: Draw the graph of $y = x^3 - 3x^2 - x + 3$.

A2.1.5 Find the zeros of a function.

Example: In the last example, find the zeros of the function; i.e., find x when $y = 0$.

A2.1.6 Solve an inequality by examining the graph.

Example: Find the solution for $x^3 - 3x^2 - x + 3 < 0$ by graphing $y = x^3 - 3x^2 - x + 3$.

A2.1.7 Graph functions defined piece-wise.

Example: Sketch the graph of $f(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x > 0 \end{cases}$.

A2.1.8 Interpret given situations as functions in graphs, formulas, and words.

Example: You and your parents are going to Boston and want to rent a car at Logan International Airport on a Monday morning and drop the car off in downtown Providence, R.I., on the following Wednesday. Find the rates from two national car companies and plot the costs on a graph. Decide which company offers the best deal. Explain your answer.

Standard 2

Linear and Absolute Value Equations and Inequalities

Students solve systems of linear equations and inequalities and use them to solve word problems. They model data with linear equations.

A2.2.1 Graph absolute value equations and inequalities.

Example: Draw the graph of $y = 2x - 5$ and use that graph to draw the graph of $y = |2x - 5|$.

A2.2.2 Use substitution, elimination, and matrices to solve systems of two or three linear equations in two or three variables.

Example: Solve the system of equations: $x - 2y + 3z = 5, x + 3z = 11, 5y - 6z = 9$.

A2.2.3 Use systems of linear equations and inequalities to solve word problems.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

A2.2.4 Find a linear equation that models a data set using the median fit method and use the model to make predictions.

Example: You light a candle and record its height in centimeters every minute. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.5), (3, 14.8), (4, 13.2), (5, 11.5), (6, 10.0), (7, 8.2), (9, 4.9), and (10, 3.1). Find the median fit line to express the candle's height as a function of the time and state the meaning of the slope in terms of the burning candle.

Standard 3

Quadratic Equations and Functions

Students solve quadratic equations, including the use of complex numbers. They interpret maximum and minimum values of quadratic functions. They solve equations that contain square roots.

A2.3.1 Define complex numbers and perform basic operations with them.

Example: Multiply $7 - 4i$ and $10 + 6i$.

A2.3.2 Understand how real and complex numbers are related, including plotting complex numbers as points in the plane.

Example: Plot the points corresponding to $3 - 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the other two?

A2.3.3 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

A2.3.4 Graph quadratic functions. Apply transformations to quadratic functions. Find and interpret the zeros and maximum or minimum value of quadratic functions.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

A2.3.5 Solve word problems using quadratic equations.

Example: You have 100 feet of fencing to make three sides of a rectangular area using an existing straight fence as the fourth side. Construct a formula in a spreadsheet to determine the area you can enclose and use the spreadsheet to make a conjecture about the maximum area possible. Prove (or disprove) your conjecture by solving an appropriate quadratic equation.

A2.3.6 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x+9} = 9 - \sqrt{x}$.

A2.3.7 Solve pairs of equations, one quadratic and one linear or both quadratic.

Example: Solve the system of equations $y = x^2 - 5x + 1, x + y + 2 = 0$.

Standard 4

Conic Sections

Students write equations of conic sections and draw their graphs.

A2.4.1 Write the equations of conic sections (circle, ellipse, parabola, and hyperbola).

Example: Write an equation for a parabola with focus $(2, 3)$ and directrix $y = 1$.

A2.4.2 Graph conic sections.

Example: Graph the circle described by the equation $(x + 4)^2 + (y - 1)^2 = 9$.

Standard 5

Polynomials

Students use the binomial theorem, divide and factor polynomials, and solve polynomial equations.

A2.5.1 Understand the binomial theorem and use it to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

A2.5.2 Divide polynomials by others of lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

A2.5.3 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

A2.5.4 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

A2.5.5 Use polynomial equations to solve word problems.

Example: You want to make an open-top box with a volume of 500 square inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Find the possible dimensions of the box.

A2.5.6 Write a polynomial equation given its solutions.

Example: Write an equation that has solutions $x = 2, x = 5i$ and $x = -5i$.

A2.5.7 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

Standard 6

Algebraic Fractions

Students use negative and fractional exponents. They simplify algebraic fractions and solve equations involving algebraic fractions. They solve problems of direct, inverse, and joint variation.

A2.6.1 Understand and use negative and fractional exponents.

Example: Simplify $(2a^{-2}b^3)^4 (4a^3b^{-1})^{-2}$.

A2.6.2 Add, subtract, multiply, divide, and simplify algebraic fractions.

Example: Simplify $\frac{x^2 - 4}{x^5} \div \frac{x^3 - 8}{x^8}$.

A2.6.3 Simplify complex fractions.

Example: Simplify $\left(\frac{5}{x-2} + \frac{2}{x+3}\right) \div \left(\frac{1}{x+3} + \frac{7}{x-2}\right)$.

A2.6.4 Solve equations involving algebraic fractions.

Example: Solve $\frac{10}{n} + \frac{5}{n^2 - 4} = \frac{7}{n - 2}$.

A2.6.5 Solve word problems involving fractional equations.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together?

A2.6.6 Solve problems of direct, inverse, and joint variation.

Example: One day your drive to work takes 10 minutes and you average 30 mph. The next day the drive takes 15 minutes. What is your average speed that day?

Standard 7

Logarithmic and Exponential Functions

Students graph exponential functions and relate them to logarithms. They solve logarithmic and exponential equations and inequalities. They solve word problems using exponential functions.

A2.7.1 Graph exponential functions.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

A2.7.2 Prove simple laws of logarithms.

Example: Use the fact that $a^x \bullet a^y = a^{x+y}$ to show that $\log_a(pq) = \log_a p + \log_a q$.

A2.7.3 Understand and use the inverse relationship between exponents and logarithms.

Example: Find the value of $\log_{10}(10^7)$.

A2.7.4 Solve logarithmic and exponential equations and inequalities.

Example: Solve the equation $\log_2 x = 5$.

A2.7.5 Use the definition of logarithms to convert logarithms from one base to another.

Example: Write $\log_{10} 75$ as a logarithm to base 2.

A2.7.6 Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values.

Example: Simplify $\log_3 81$.

A2.7.7 Use calculators to find decimal approximations of natural and common logarithmic numeric expressions.

Example: Find a decimal approximation for ln 500.

A2.7.8 Solve word problems involving applications of exponential functions to growth and decay.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

Standard 8

Sequences and Series

Students define and use arithmetic and geometric sequences and series.

A2.8.1 Define arithmetic and geometric sequences and series.

Example: What type of sequence is 10, 100, 1,000, 10,000, …?

A2.8.2 Find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15, … .

A2.8.3 Find partial sums of arithmetic and geometric series.

Example: In the last example, find the sum of the first 10 terms.

A2.8.4 Solve word problems involving applications of sequences and series.

Example: You have on a Petri dish 1 square millimeter of a mold that doubles in size each day. What area will it cover after a month?

Standard 9

Counting Principles and Probability

Students use fundamental counting principles to compute combinations, permutations, and probabilities.

A2.9.1 Understand and apply counting principles to compute combinations and permutations.

Example: There are 5 students who work in a bookshop. If the bookshop needs 3 people to operate, how many days straight could the bookstore operate without the same group of students working twice?

A2.9.2 Use the basic counting principle, combinations, and permutations to compute probabilities.

Example: You are on a chess team made up of 15 players. What is the probability that you will be chosen if a 3-person team is selected at random?

Standard 10

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

A2.10.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: The swimming pool at Roanoke Park is 24 feet long and 18 feet wide. The park district has determined that they have enough money to put a walkway of uniform width, with a maximum area of 288 square feet, around the pool. How could you find the maximum width of a new walkway?

A2.10.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 20 feet. Is that reasonable?

Students develop and evaluate mathematical arguments and proofs.

A2.10.3 Decide if a given algebraic statement is true always, sometimes, or never (statements involving rational or radical expressions or logarithmic or exponential functions).

Example: Is the statement $(a^x)^y = a^{xy}$ true for all x, for some x, or for no x?

A2.10.4 Use the properties of number systems and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Simplify $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$, explaining why you can take each step.

A2.10.5 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: A student solving the equation $\sqrt{x + 6} = x$ comes up with the solution set $\{-2, 3\}$. Explain why $\{-2, 3\}$ is not the solution set to this equation, and why the "check" step is essential in solving the equation.

A2.10.6 Use counterexamples to show that statements are false.

Example: Show by an example that this statement is false: The product of two complex numbers is never a real number.

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