

An agricultural field trial compares the yield of two varieties of tomato for commercial use. The researchers divide in half each of 10 small plots of land in different locations and plant each tomato variety on one half of each plot. After harvest, they compare the yields in pounds per plant at each location. The 10 differences (Variety A – Variety B) give $\bar{x} = 0.34$. Assume a population standard deviation of $\sigma = 0.83$.

We want to determine if there is a difference in the mean yields, so our null hypothesis is that there is no difference in the mean yields, i.e., $H_0: \mu = 0$. $H_a: \mu \neq 0$. We asked, “Is there convincing evidence at the 5% significance level that the mean yield of tomatoes is different from $\mu = 0$?” A significance test to answer this question was carried out. The test statistic is $z = 1.2954$, and the P -value is 0.1952.

1. Describe what a Type I error would be in this situation.
2. Calculate the probability of a Type I error for this problem.
3. The researchers who carried out the field trial suspect that the large P -value is due to low power. They would like to be able to detect a mean difference in yield of 0.5 pound per plant at the $\alpha = 0.05$ significance level. Find the probability of a Type II error when in fact $\mu = 0.5$.
Then find the power of the test against the alternative $\mu = 0.5$.
4. If the sample size is increased to $n = 25$ plots of land, what will be the power against the same alternative?