

Sec 2.1

## Success Criteria

- Construct a cumulative relative frequency graph (Ogive)
- Interpret an Ogive
- Determine the effect of changing units on summary measures

## Percent vs Percentile

1) Take test, get 75%

Had there been 100 questions, you would have gotten 75 correct

2) Take test, score in 75th percentile

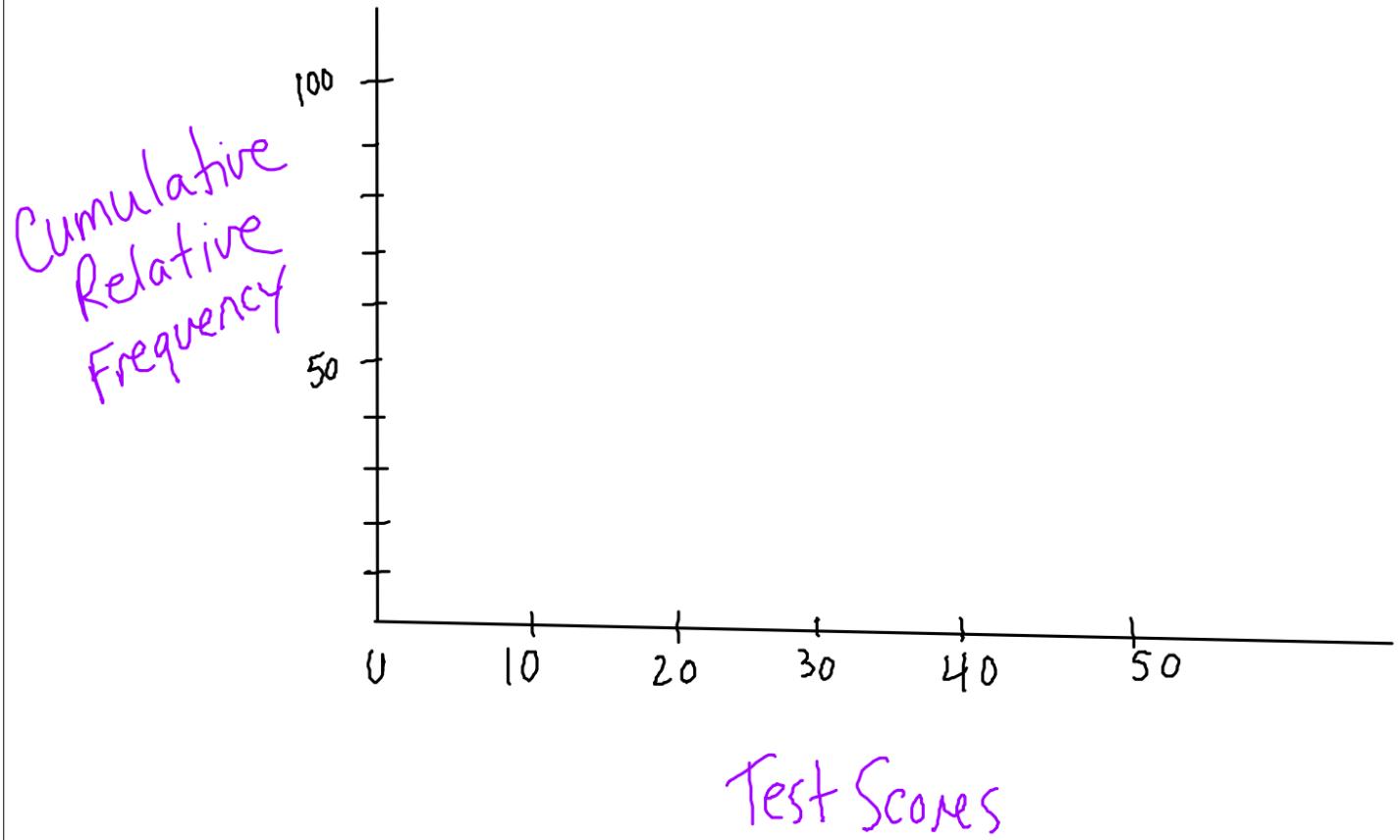
You did better than 75% of everyone who took the test (actual score unknown)

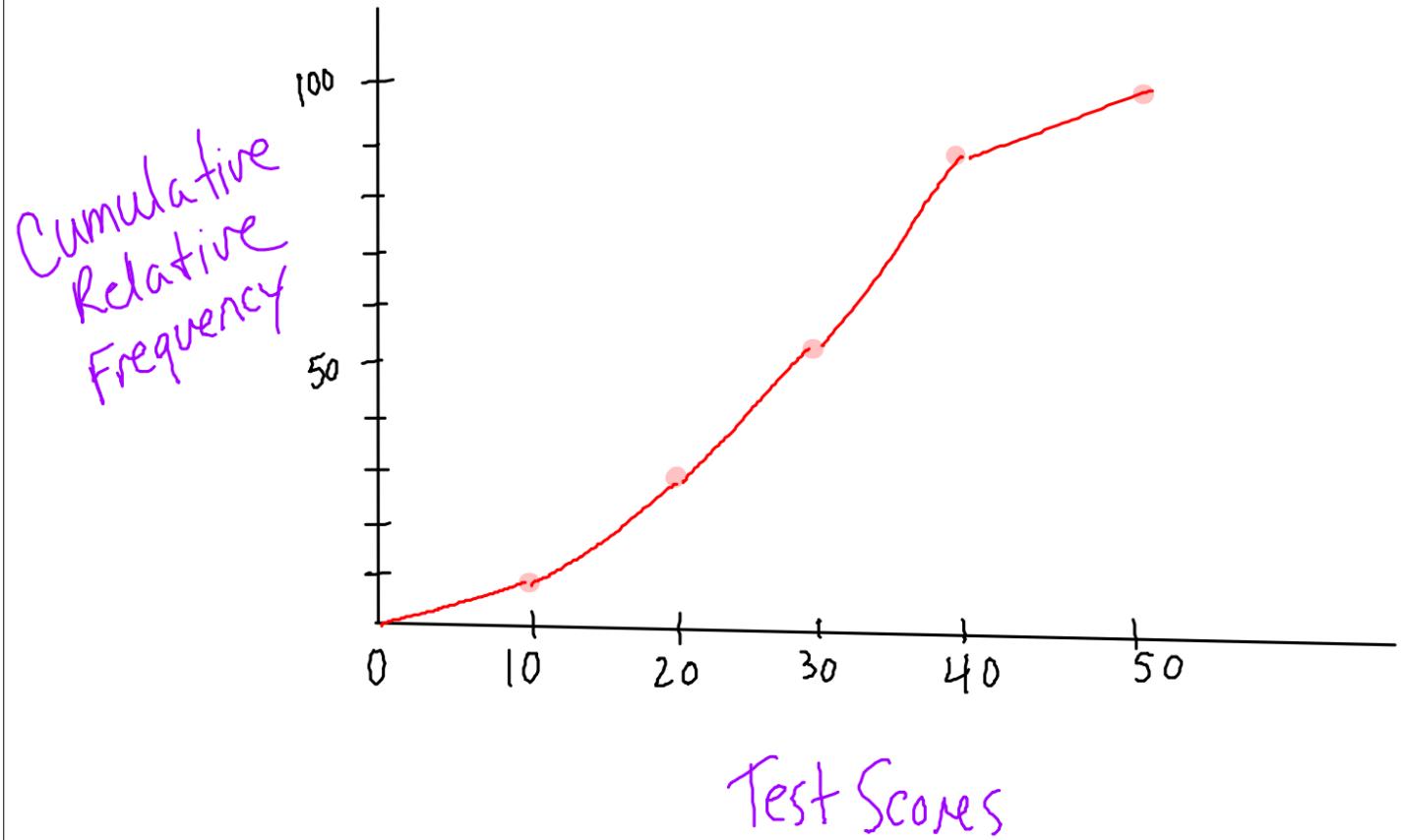
## Percentile

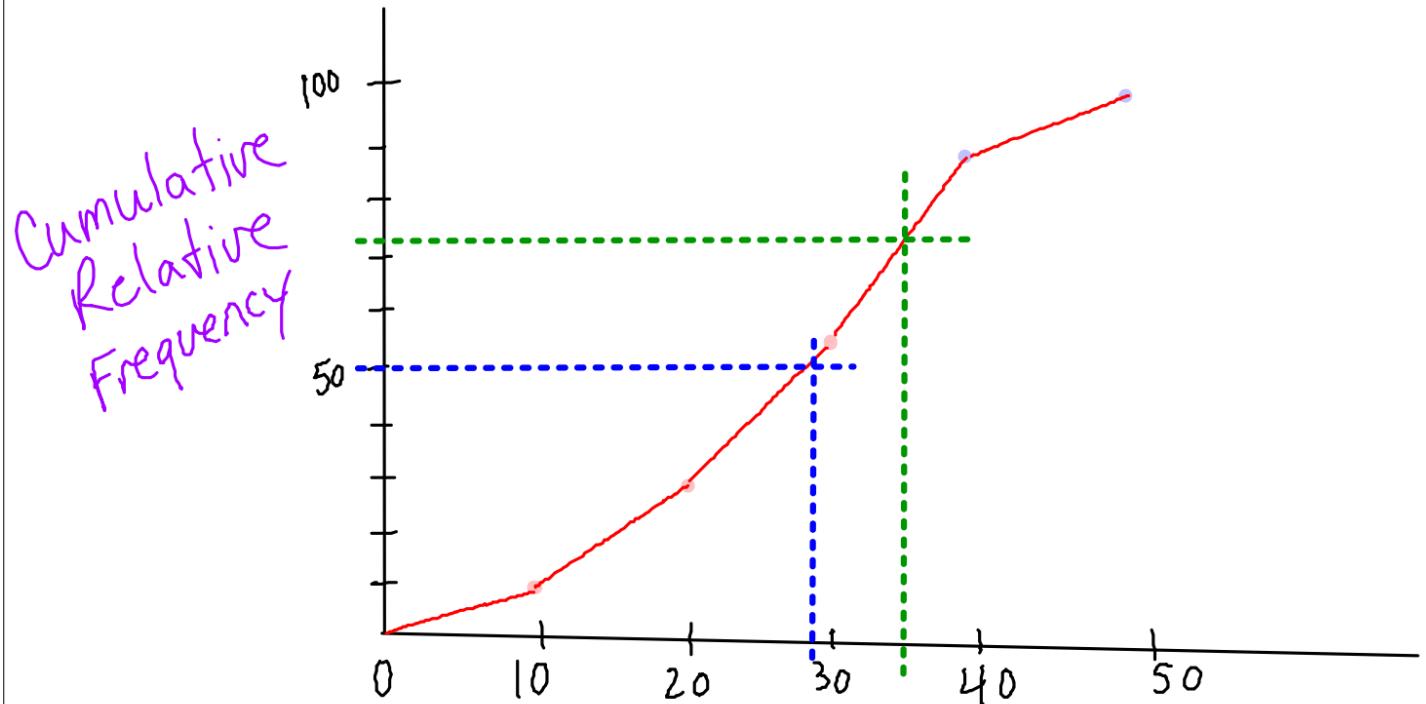
The  $p$ th percentile of a distribution is the value with  $p$  percent of the observations less than it

# Ogives (Cumulative Relative Frequency Graphs)

Test Score	Frequency	Relative Frequency	Cumulative Relative Frequency
0 - 9	4	10%	10%
10 - 19	8	20	30
20 - 29	10	25	55
30 - 39	15	37.5	92.5
40 - 50	3	7.5	100
	<u>40</u>	<u>100%</u>	







Test Scores

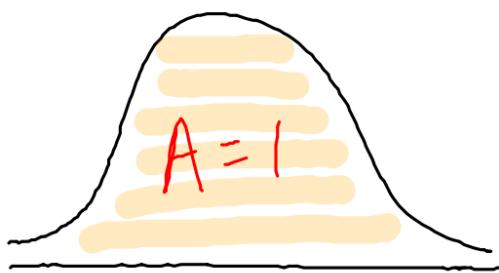
50th Percentile  $\approx \underline{28}$

$35 \approx \underline{72}$  Percentile

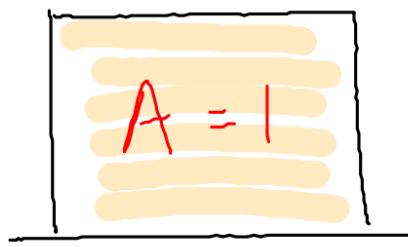
## Density Curves

Data → Graph (Histogram) → Smooth Curve → Model

## Symmetric Distributions (Mean = Median)

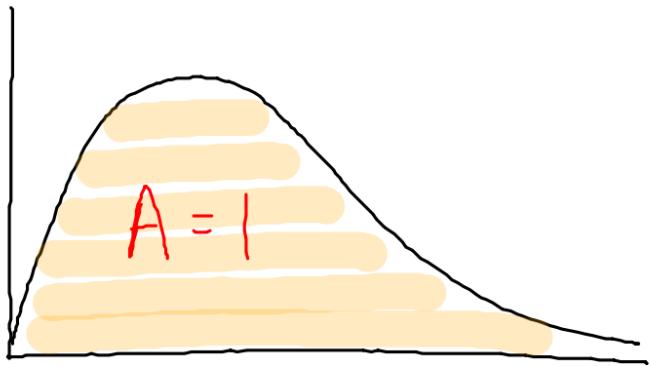


Bell Curve

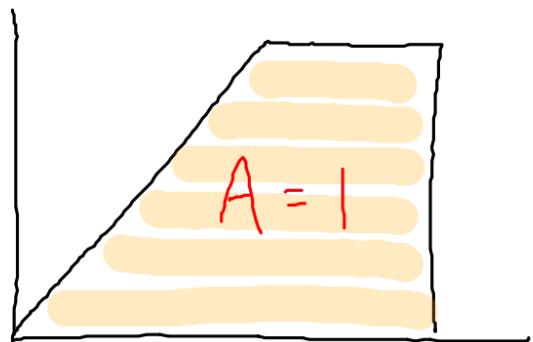


Uniformed

# Skewed Distributions



Skewed Right  
(Mean > Median)



Skewed Left  
(Mean < Median)

# Transforming Data

Data Set

$$\begin{aligned} & \times 5 \quad \left( \begin{array}{l} \{1, 2, 3\} \\ \{5, 6, 7\} \end{array} \right) + 4 \\ & \rightarrow \{5, 10, 15\} \end{aligned}$$

$\bar{x}$

2
6
10

$s$

1
1
5

Means effected by  
addition and multiplication

Standard deviations  
effected by multiplication  
only

Ex The average temperature of a cabin is  $8.43^{\circ}\text{C}$  with standard deviation of  $2.27^{\circ}\text{C}$

i) Find the average temp in F

$$F = \frac{9}{5}C + 32 = \frac{9}{5}(8.43) + 32 = 47.174^{\circ}$$

2) Find the standard deviation in F

$$F = \frac{9}{5}C + \cancel{32} = 4.086^{\circ}$$

Data Set

$$\{ 5, 10, 15 \}$$

$$\downarrow \frac{9}{5}x + 32$$

$$\{ 41, 50, 59 \}$$

$$\bar{x} \quad s$$

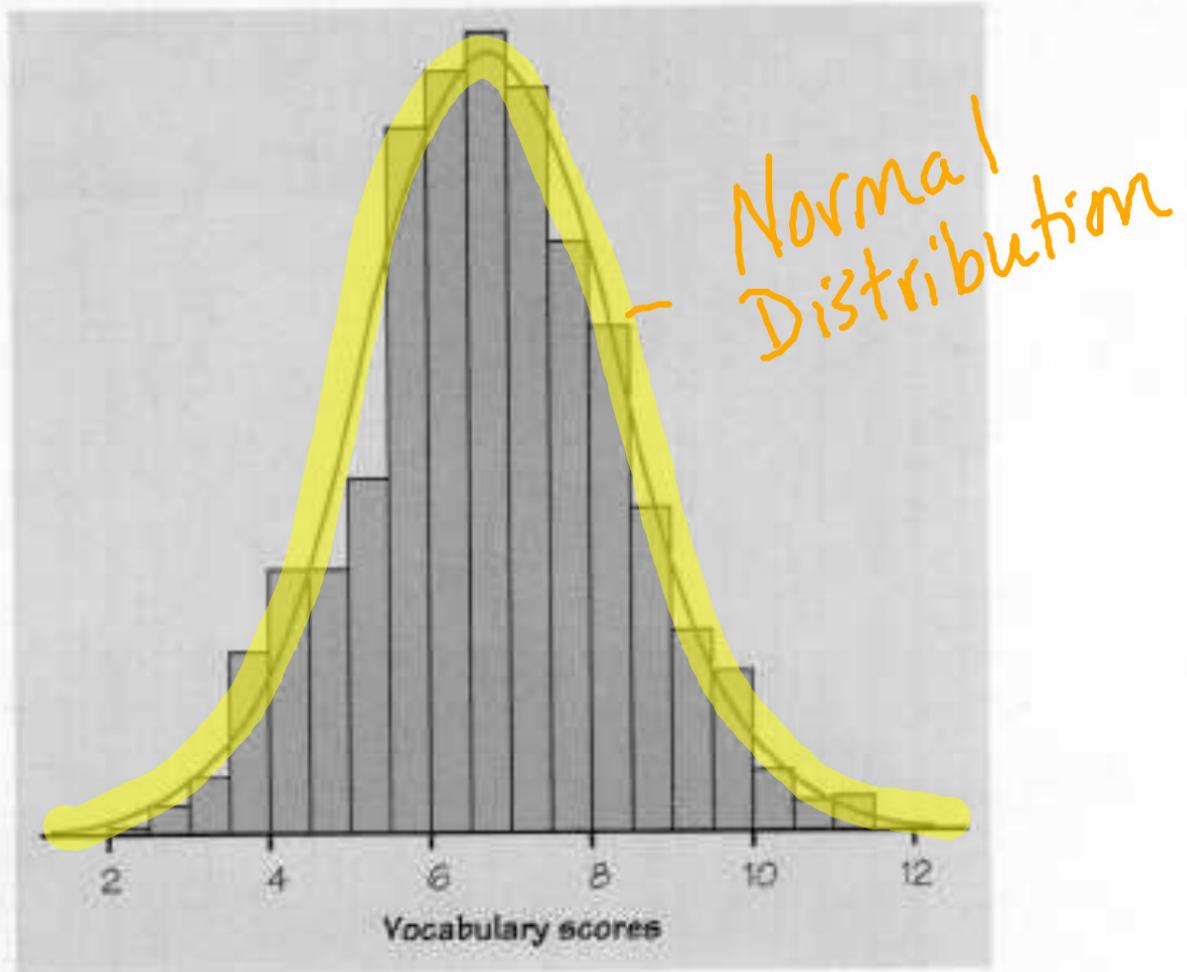
$$10 \quad 5$$

$$\left[ \frac{9}{5}(10) + 32 \right] \quad \left[ \frac{9}{5}(5) \right]$$

Sec 2.2

## Success Criteria

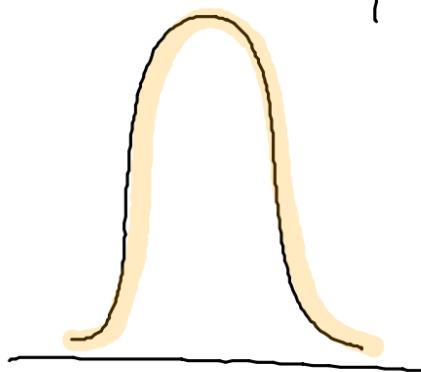
- Calculate probabilities from a Normal distribution using properties and technology
- Calculate values of percentiles
- Determine if data is Normally distributed



**FIGURE 2.2** Histogram of the vocabulary scores of all seventh-grade students in Gary, Indiana. The smooth curve shows the overall shape of the distribution.

## Normal Distributions

- Described by a normal density curve



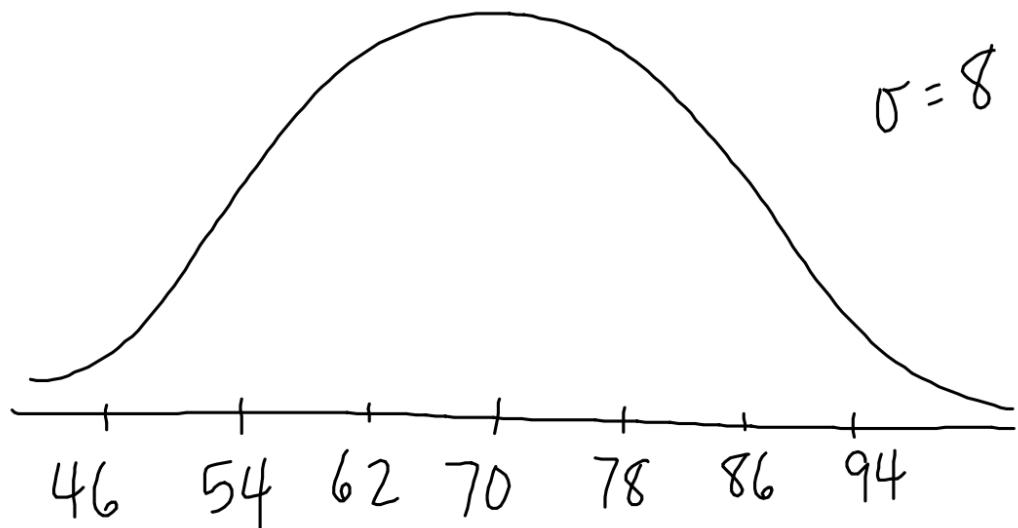
- Uses  $\mu$  (for mean) and  $\sigma$  (standard deviation)
- Notation:  $N(\mu, \sigma)$

## Note

- All Normal Distributions are symmetric  
but not all Symmetric Distributions  
are Normal

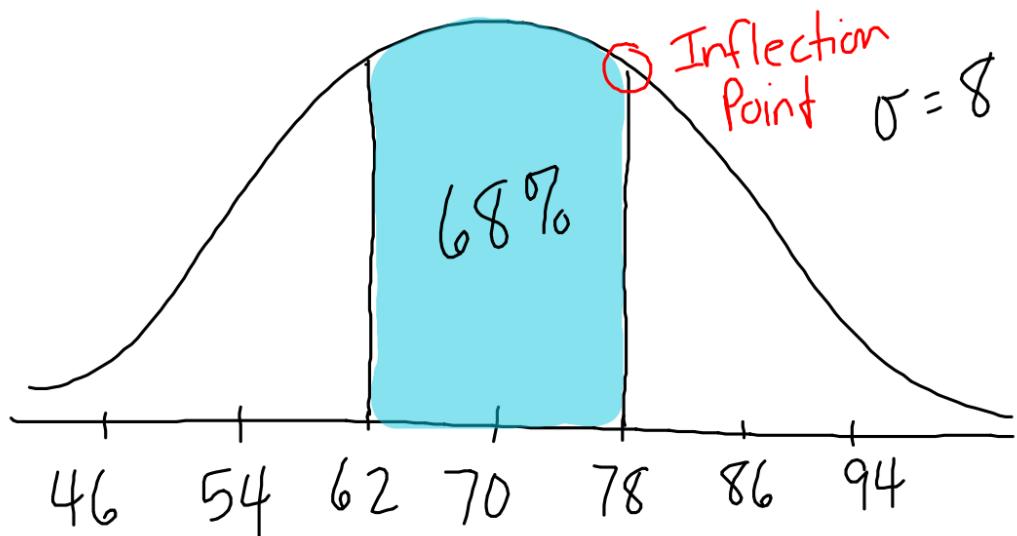
68 - 95 - 99.7 Rule

Ex Stats Exam  $\rightarrow N(70, 8)$



68 - 95 - 99.7 Rule

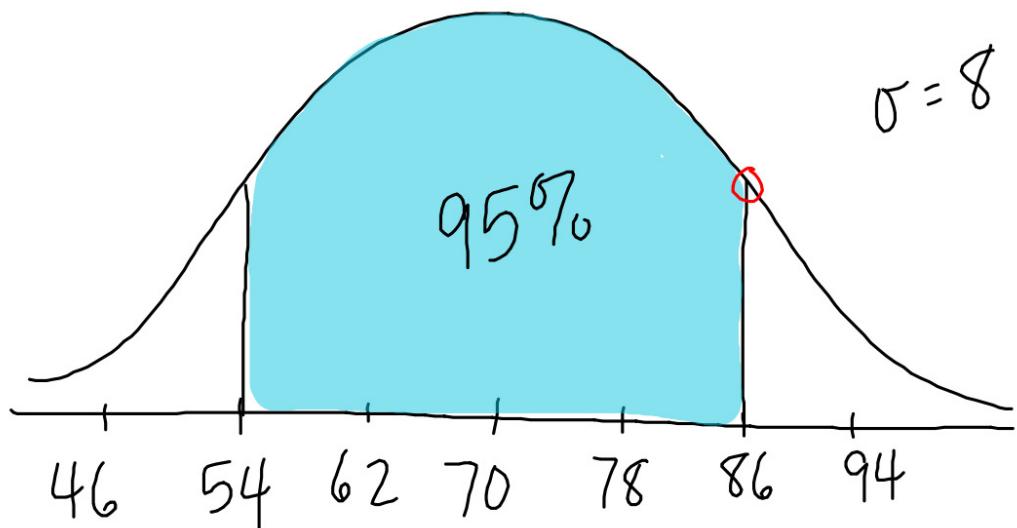
Ex Stats Exam  $\rightarrow N(70, 8)$



Aprox 68% of all test scores are within  $\pm 1\sigma$  from the mean

68 - 95 - 99.7 Rule

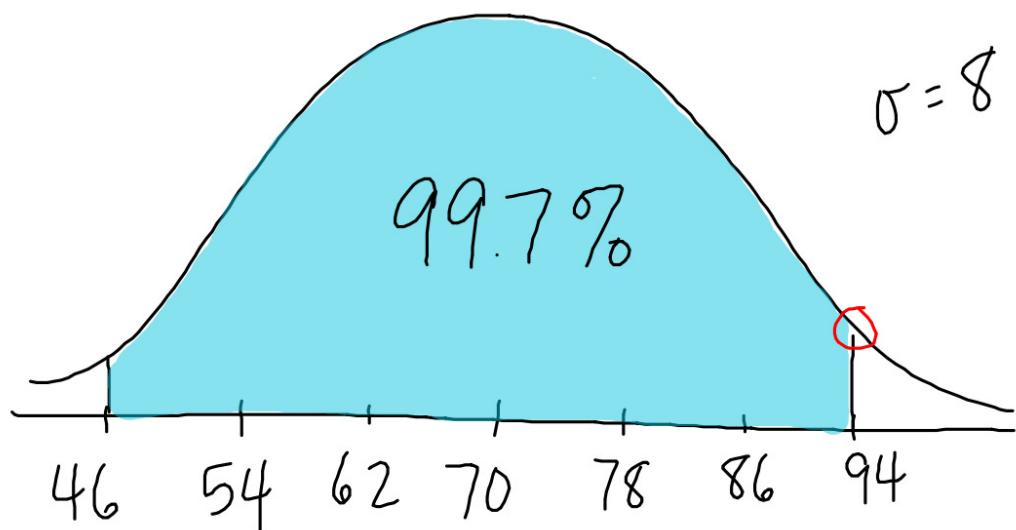
Ex Stats Exam  $\rightarrow N(70, 8)$



Aprox 95% of all test scores are within  $\pm 2\sigma$  from the mean

68 - 95 - 99.7 Rule

Ex Stats Exam  $\rightarrow N(70, 8)$

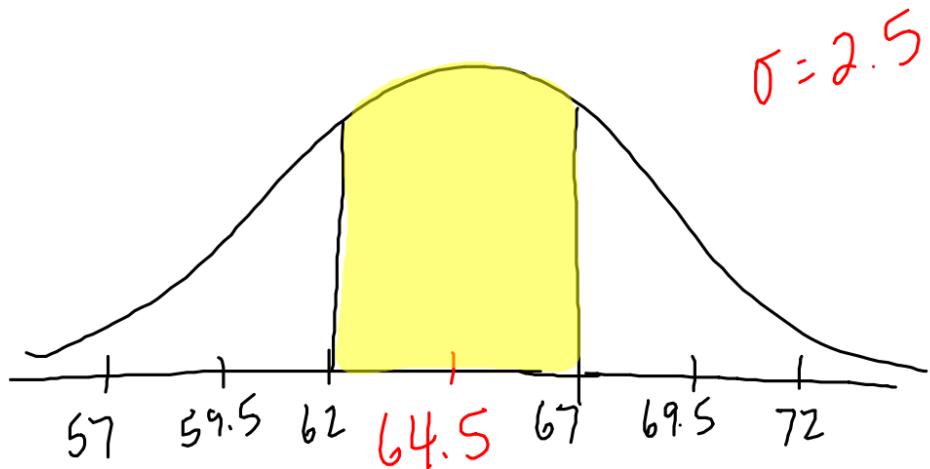


Aprox 99.7% of all test scores are within  $\pm 3\sigma$  from the mean

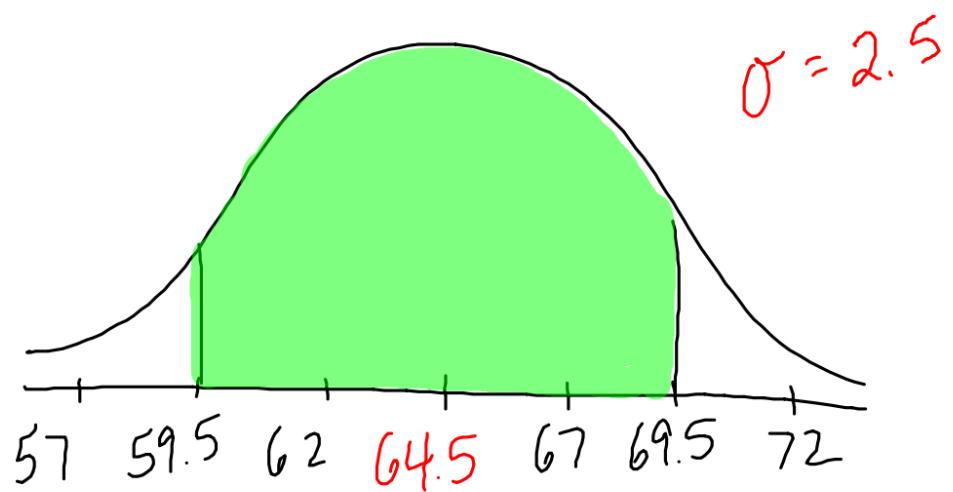
Ex Avg height women  $\rightarrow N(64.5, 2.5)$

i) 68 % women between 62" and 67"

Sketch Problem:

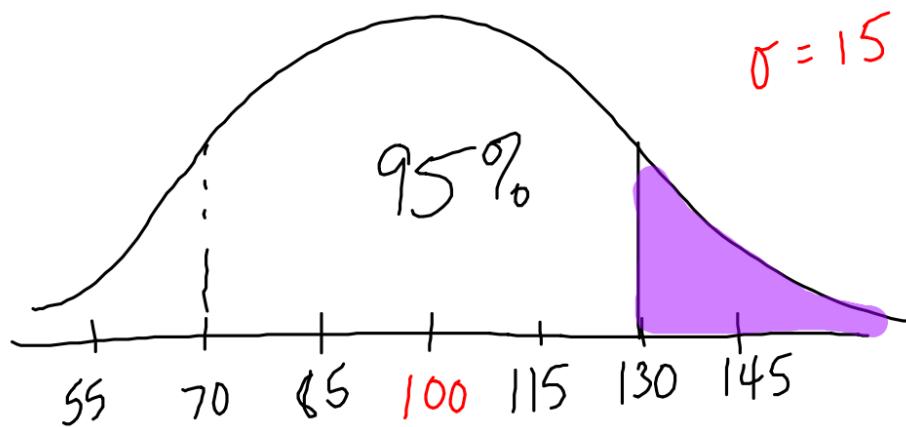


2) 95 % between 59.5 and 69.5 ?

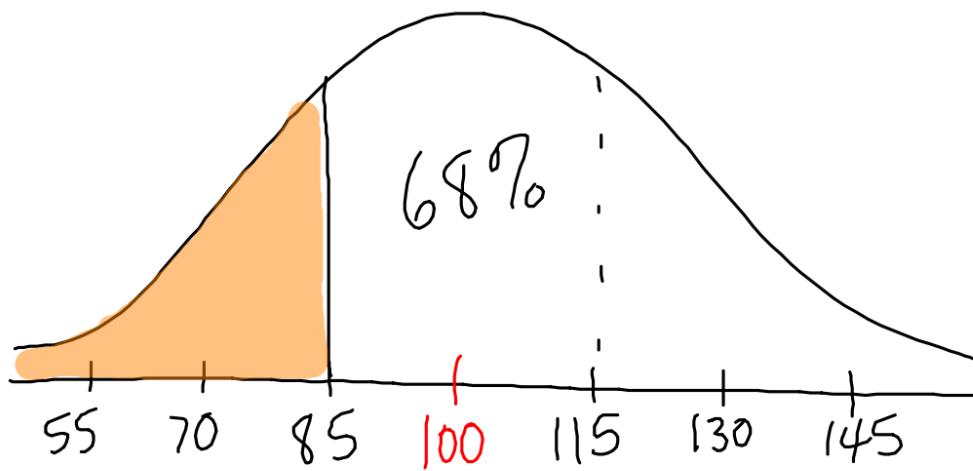


Ex IQ Scores  $\rightarrow N(100, 15)$

1) 2.5 % scores  $> 130$

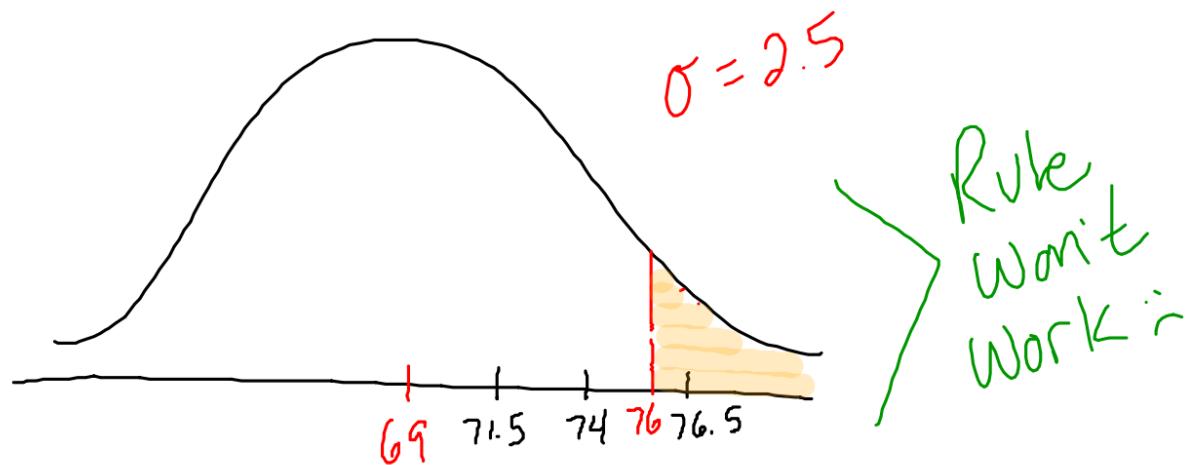


2) 16 % of scones < 85?



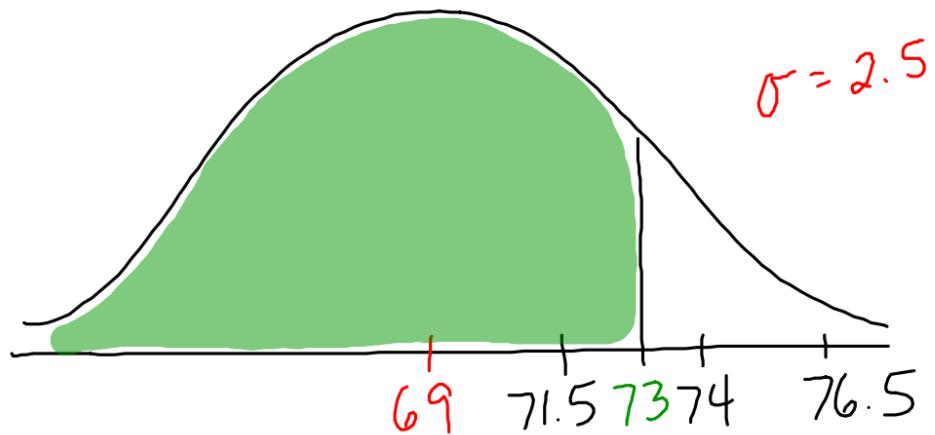
Ex Height of Men  $\rightarrow N(69, 2.5)$

1) .25 % of men  $> 76''$



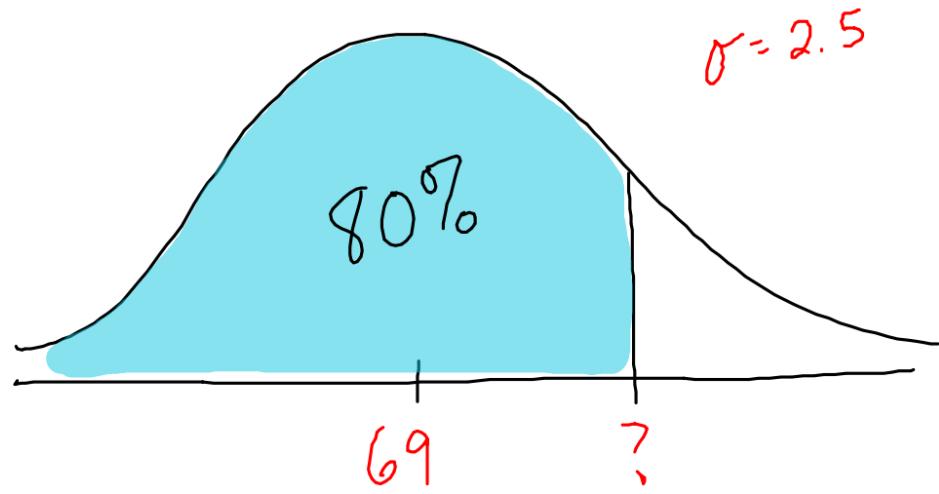
DISTR  $\rightarrow \text{normalcdf}(76, 1000, 69, 2.5) \approx .0025$   
2nd VARS      Upper Lower  $\mu$   $\sigma$

2) 94.52% of men  $\leq 73''$



$$\text{normalcdf}(0, 73, 69, 2.5) \approx .9452$$

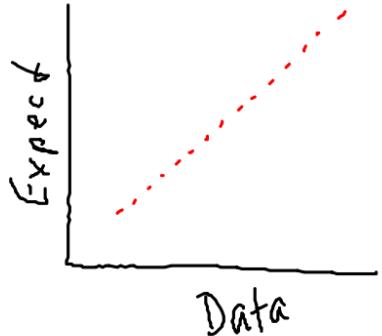
3) What height puts someone in  
the 80th percentile?



DISTR  $\rightarrow \text{invNorm} (.80, 69, 2.5) \approx 71.1''$   
Area  $\mu \sigma$

## Assessing Normality

- 1) Graph data < span style="display: inline-block; vertical-align: middle; margin-left: 10px;">roughly symmetric?  
outliers?
  - 2) Compare mean and median
  - 3) Apply 68-95-99 Rule
  - ★ 4) Use normal probability plot
- } Symmetry



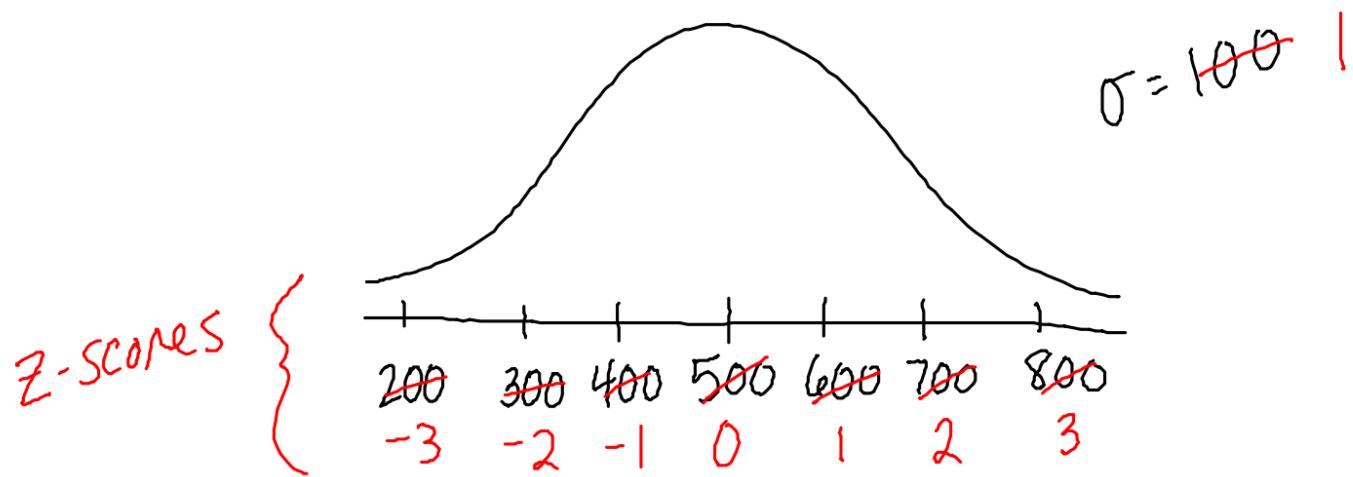
Sec 2.2 (cont)

## Success Criteria

- Calculate Z-scores from a Normal distribution
- Determine probabilities from a Normal distribution using Z-scores and a cumulative probability Table
- Calculate values of percentiles using Z-scores

## Standard Normal Curve

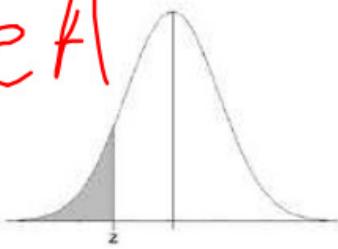
Ex SAT Scores  $\rightarrow N(500, 100)$



$$Z = \frac{X - M}{\sigma} \rightarrow \text{Table A}$$

## Standard Normal Cumulative Probability Table

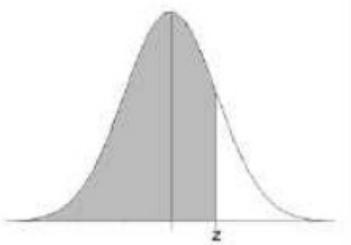
> Table A



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

<b>z</b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

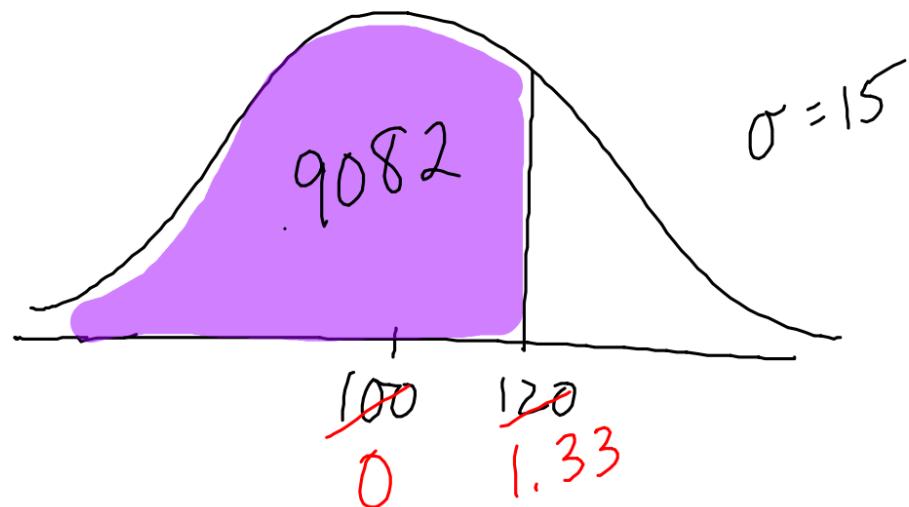
## Standard Normal Cumulative Probability Table



**Cumulative probabilities for POSITIVE z-values are shown in the following table:**

IQ Scores  $\rightarrow N(100, 15)$

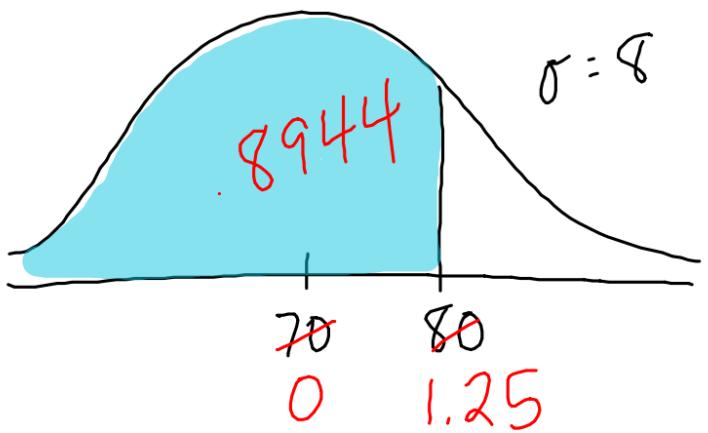
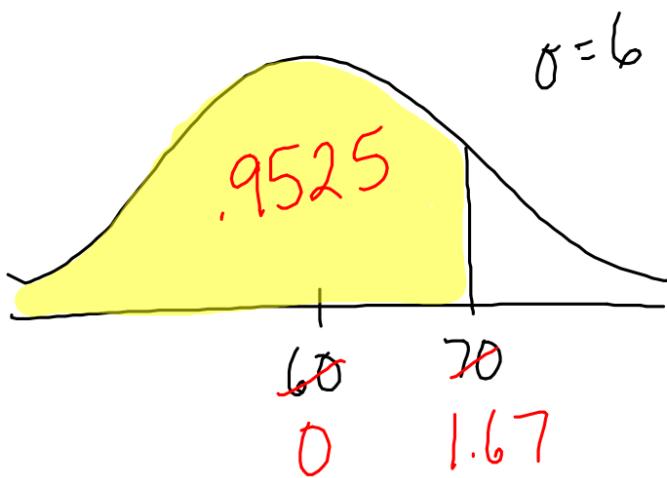
i) What proportion  $< 120$ ?



Who did better on 2 different tests?

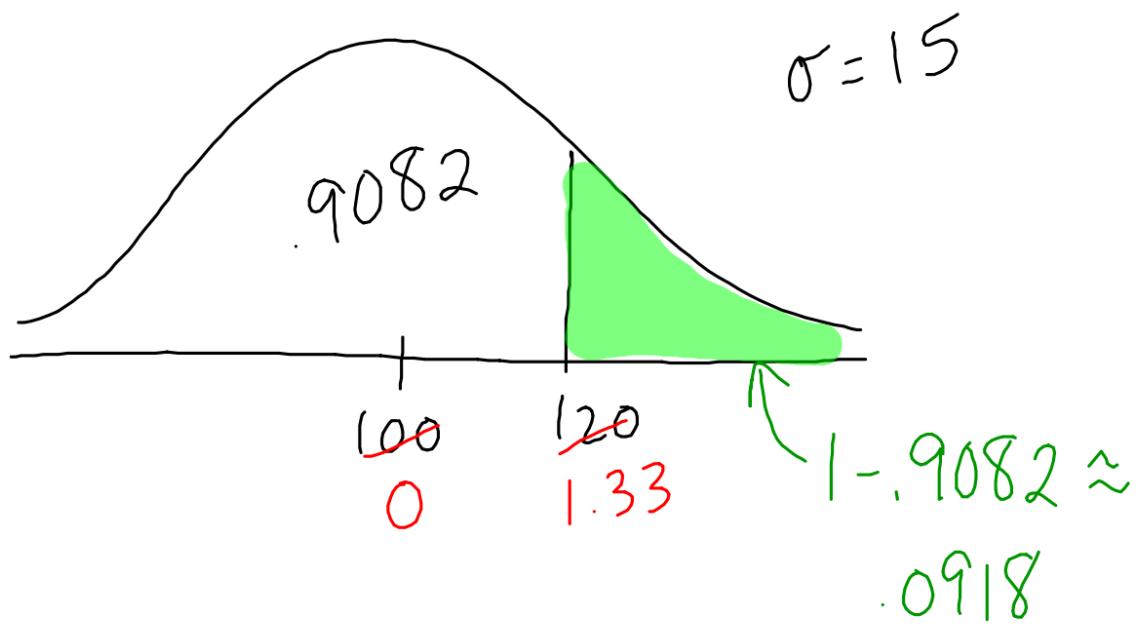
\* Mary  $\rightarrow 70/100$   
 $N(60, 6)$

John  $\rightarrow 80/100$   
 $N(70, 8)$

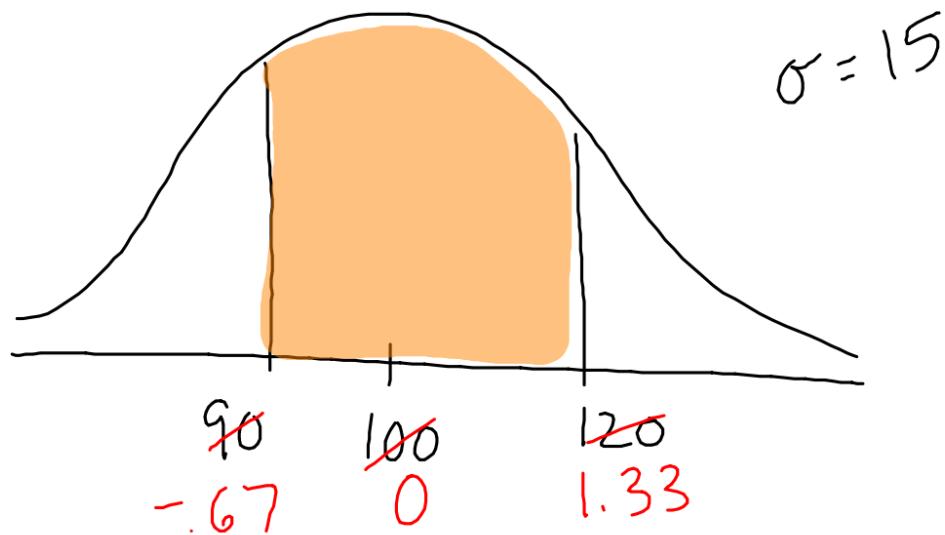


\* Mary did better - larger z score and higher percentile

2) What proportion scores  $> 120$ ?

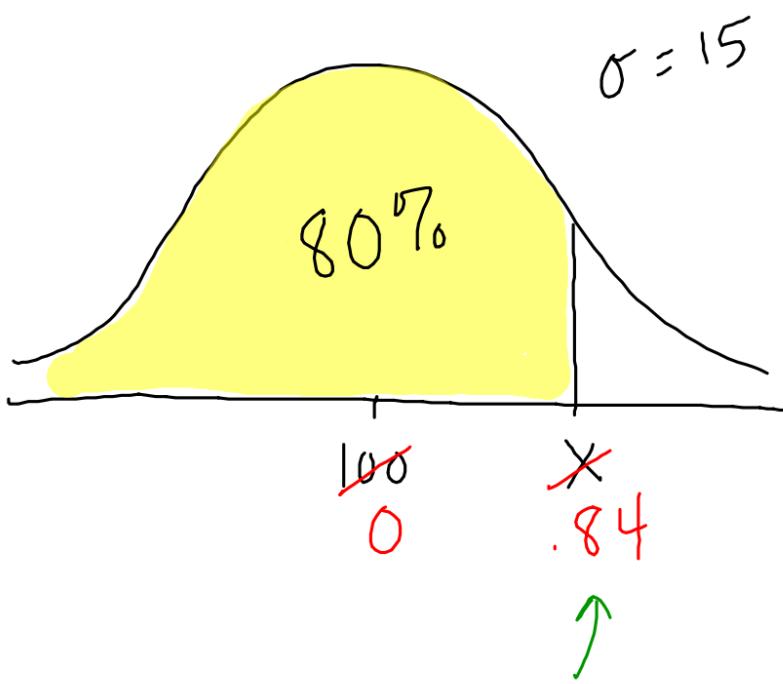


3) What % are between 90 and 120?



4) What score puts someone in 80th percentile?

4) What score puts someone in 80th percentile?



$$Z = \frac{X - \mu}{\sigma}$$

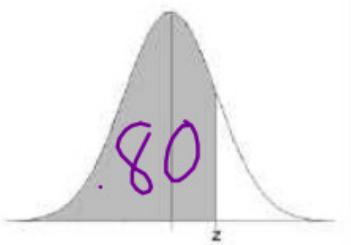
$$\frac{.84}{1} = \frac{X - 100}{15}$$

$$12.6 = X - 100$$

$$112.6 = X$$

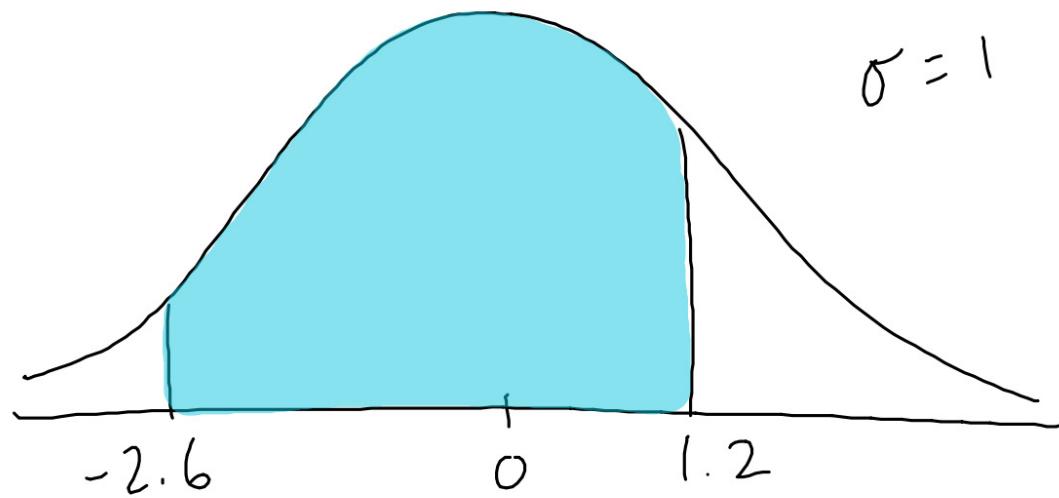
See Next  
Page

## Standard Normal Cumulative Probability Table



**Cumulative probabilities for POSITIVE z-values are shown in the following table:**

## Calculator Note



$$\text{normalcdf}(-2.6, 1.2, \cancel{0}, \cancel{1}) \approx .8803$$

## Summary (Calculations With

1) 68-95-99.7 Rule

Normalcdf



2) Calculator

[DISTR]

invNorm

3) Z-score (Table A)

