OGIVES

Marsh supermarket recorded the length of time, to the nearest minute, that a sample of 200 cars was parked in their lot. The results were:

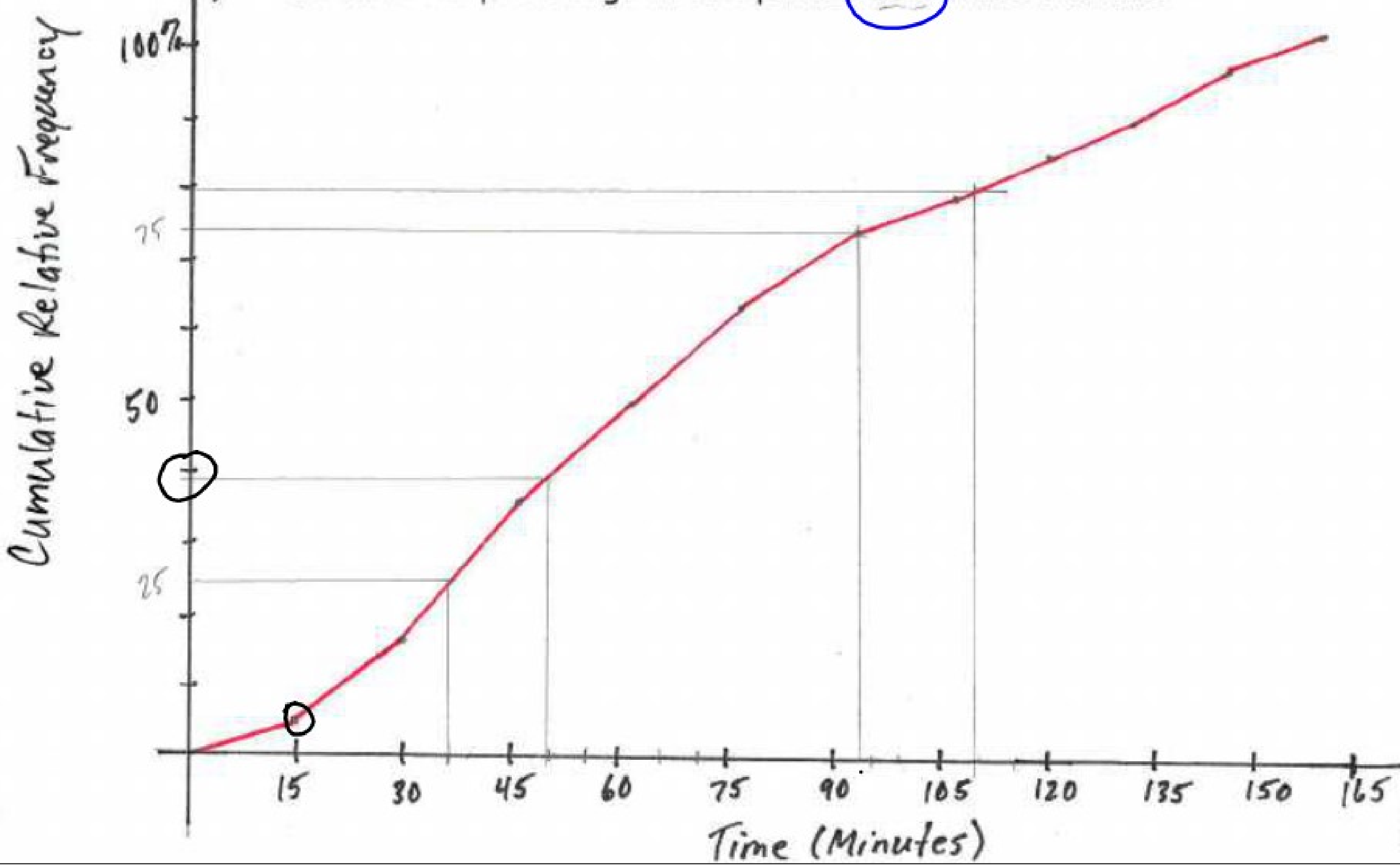
X Y

Time (minutes)		Frequency	%	Cumulative Relative Freq
0 - 14		13	6.5	6.5
15 - 29	≤56 min = 68	23	11.5	18
30 - 44		32	16	34
45 - 59	= 33	33	16.5	50.5
60 - 74		27	13.5	64
75 - 89		20	10	74
90 - 104	≤56 min = 99	12	6	80
105 - 119		11	5.5	85.5
120 - 134		10	5	90.5
135 - 149		11	5.5	96
150 - 164		8	4	100
		200	100%	

a) Draw an Ogive and use it to estimate the upper and lower quartiles 25%ile ≈ 38 min 75%ile ≈ 94 min

b) Estimate the 80th percentile 108 min

c) Estimate the percentage of cars parked more than 50 minutes 100% - 39% = 61%



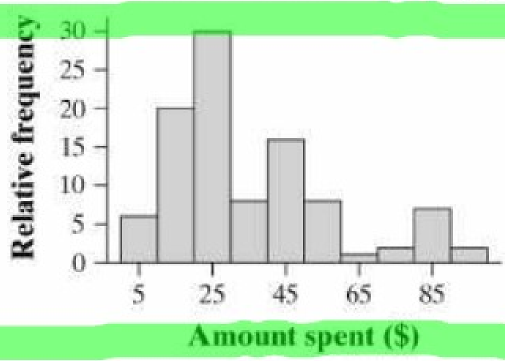
2.5 The girl in question weighs more than 48% of girls her age, but is taller than 78% of the girls her age. Since she is taller than 78% of girls, but only weighs more than 48% of girls, she is probably fairly skinny.

2.6 Peter's time was slower than 80% of his previous race times that season, but it was slower than only 50% of the racers at the league championship meet.

2.7 (a) The highlighted student sent about 212 text messages in the 2-day period which placed her at about the 80^{th} percentile. (b) The median number of texts is the same as the 50^{th} percentile. Locate 50% on the y-axis, read over to the points and then find the relevant place on the x-axis. The median is approximately 115 text messages.

2.8 (a) Maryland has about 13% foreign-born residents placing it at about the 70^{th} percentile. (b) Locate 30% on the y-axis, read over to the points and then find the relevant place on the x-axis. The 30^{th} percentile is approximately 4.5% foreign-born.

2.9 (a) First find the quartiles. The first quartile is the 25^{th} percentile. Find 25 on the y-axis, read over to the line and then down to the x-axis to get about \$19. The 3^{rd} quartile is the 75^{th} percentile. Find 75 on the y-axis, read over to the line and then down to the x-axis to get about \$48 So the interquartile range is \$48 – \$19 = \$29. (b) The person who spent \$19.50 is just above what we have called the 25^{th} percentile. It appears that \$19.50 is at about the 26^{th} percentile. (c) The graph is below:



2.10 (a) To find the 60^{th} percentile, find 60 on the y-axis, read over to the line and then read down to the x-axis to find approximately 1000 hours. (b) To find the percentile for the lamp that lasted 900 hours, find 900 on the x-axis, read up to the line and across to the y-axis to find that it is approximately the 35^{th} percentile.

2.11 Eleanor's standardized score, $z = \dfrac{680-500}{100} = 1.8$, is higher than Gerald's standardized score, $z = \dfrac{27-18}{6} = 1.5$.

he did among the 50 boys at his school. (b) Scott's z-scores are $z = \frac{64 - 46.9}{10.9} = 1.57$ among the national group and $z = \frac{64 - 58.2}{9.4} = 0.62$ among the 50 boys at his school.

2.18 The boys at Scott's school did very well on the PSAT. Scott's score was relatively better when compared to the national group than to his peers at school. Only 5.2% of the test takers nationally scored 65 or higher, yet about 32% scored 65 or higher at Scott's school.

2.19 (a) The mean and the median both increase by 18 so the mean is 87.188 and the median is 87.5.

$$\text{Mean}_{\text{Now}} = \frac{\text{sum of student heights standing on chairs}}{\text{number of students}}$$

$$= \frac{(\text{height of first student} + 18) + \ldots + (\text{height of last student} + 18)}{\text{number of students}}$$

$$= \frac{\text{sum of student heights standing on floor} + 18 * \text{number of students}}{\text{number of students}}$$

$$= \text{Mean}_{\text{Old}} + 18 = 69.188 + 18 = 87.188.$$

The median is still the height of the middle student. Now that this student is standing on a chair 18 inches from the ground, the median will be 18 inches larger. (b) The standard deviation and *IQR* do not change. For the standard deviation, note that although the mean increased by 18, the observations each increased by 18 as well so that the deviations did not change. For the *IQR*, Q_1 and Q_3 both increase by 18 so that their difference remains the same as in the original data set.

2.20 (a) The mean and median salaries will each increase by \$1000 (the distribution of salaries just shifts by \$1000). (b) The extremes and quartiles will also each increase by \$1000. The standard deviation will not change. Nothing has happened to affect the variability of the distribution. The center has shifted location, but the spread has not changed.

2.21 (a) To give the heights in feet, not inches, we would divide each observation by 12 (12 inches = 1 foot). Thus

$$\text{Mean}_{\text{New}} = \frac{\frac{\text{height of first student (inches)}}{12} + \ldots + \frac{\text{height of last student(inches)}}{12}}{\text{number of students}}$$

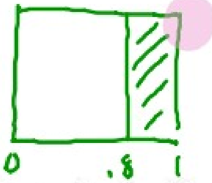
$$= \frac{1}{12}\left(\frac{\text{height of first student (inches)} + \ldots + \text{height of last student (inches)}}{\text{number of students}}\right)$$

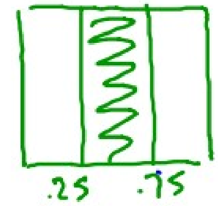
$$= \frac{1}{12}\text{Mean}_{\text{Old}} = \left(\frac{1}{12}\right)69.188 = 5.77 \text{ feet.}$$

The median is still the height of the middle student. To convert this height to feet, we divide by 12:

$$\text{Median}_{\text{New}} = \frac{69.5}{12} = 5.79 \text{ feet.}$$

b)





so the proportion is $\frac{1}{3}$. (c) One-tenth of accidents occur next to Sue's property: this is a $\frac{1}{3}$ by 0.3 rectangle, so the proportion is 0.1.

2.28 (a) The area under the curve is a rectangle with height 1 and width 1. Thus, the total area under the curve is $1(1)=1$. (b) The area under the uniform distribution between 0.8 and 1 is $0.2(1)=0.2$, so 20% of the observations lie above 0.8. (c) The area under the uniform distribution between 0.25 and 0.75 is $0.5(1)=0.5$, so 50% of the observations lie between 0.25 and 0.75.

2.29 Both are 1.5. The mean is 1.5 because this is the obvious balance point of the rectangle. The median is also 1.5 because the distribution is symmetric (so that median = mean) and because half of the area lies to the left and half to the right of 1.5.

2.30 Both are 0.5. The mean is 0.5 because this is the obvious balance point of the rectangle. The median is also 0.5 because the distribution is symmetric (so that median = mean) and because half of the area lies to the left and half to the right of 0.5.

2.31 (a) Mean is C, median is B (the right skew pulls the mean to the right). (b) Mean is B, median is B (this distribution is symmetric).

2.32 (a) Mean is A, median is A (the distribution is symmetric). (b) Mean A, median B (the left skew pulls the mean to the left).

2.33 c

2.34 b

2.35 c

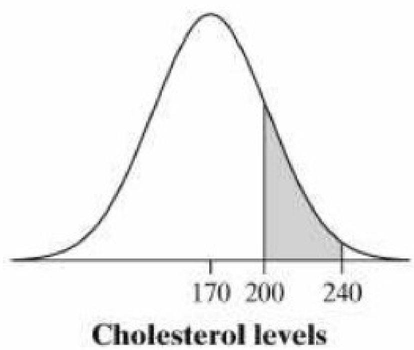
2.36 b

2.37 d

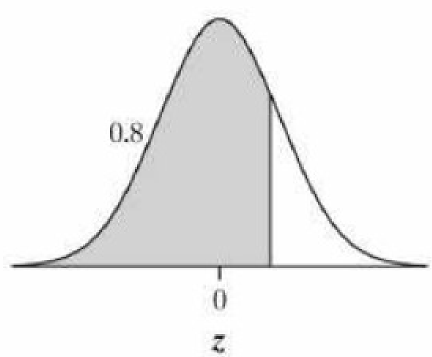
2.38 e

2.39 The distribution is skewed to the right since most of the values are 25 minutes or less, but the values stretch out up to about 90 minutes. The data is centered roughly around 20 minutes and the range of the distribution is close to 90 minutes. The two largest values appear to be outliers.

2. The z-score for a cholesterol level of 200 is $z = \frac{200-170}{30} = 1$. The proportion of z-scores between 1 and 2.33 is $0.9901 - 0.8413 = 0.1488$. A graph is shown below:



3. The 80th percentile of the Standard Normal distribution is 0.84 (see graph below). This means that the distance, x, of Tiger Woods' drive lengths that satisfies the 80th percentile is the solution to $0.84 = \frac{x-304}{8}$. Solving for x, we get 310.72 yards.



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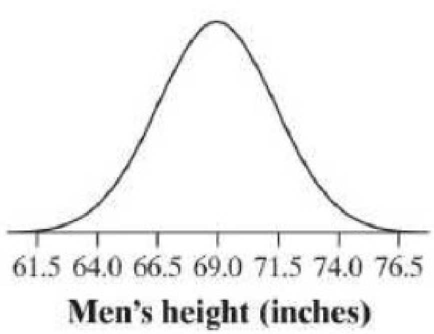
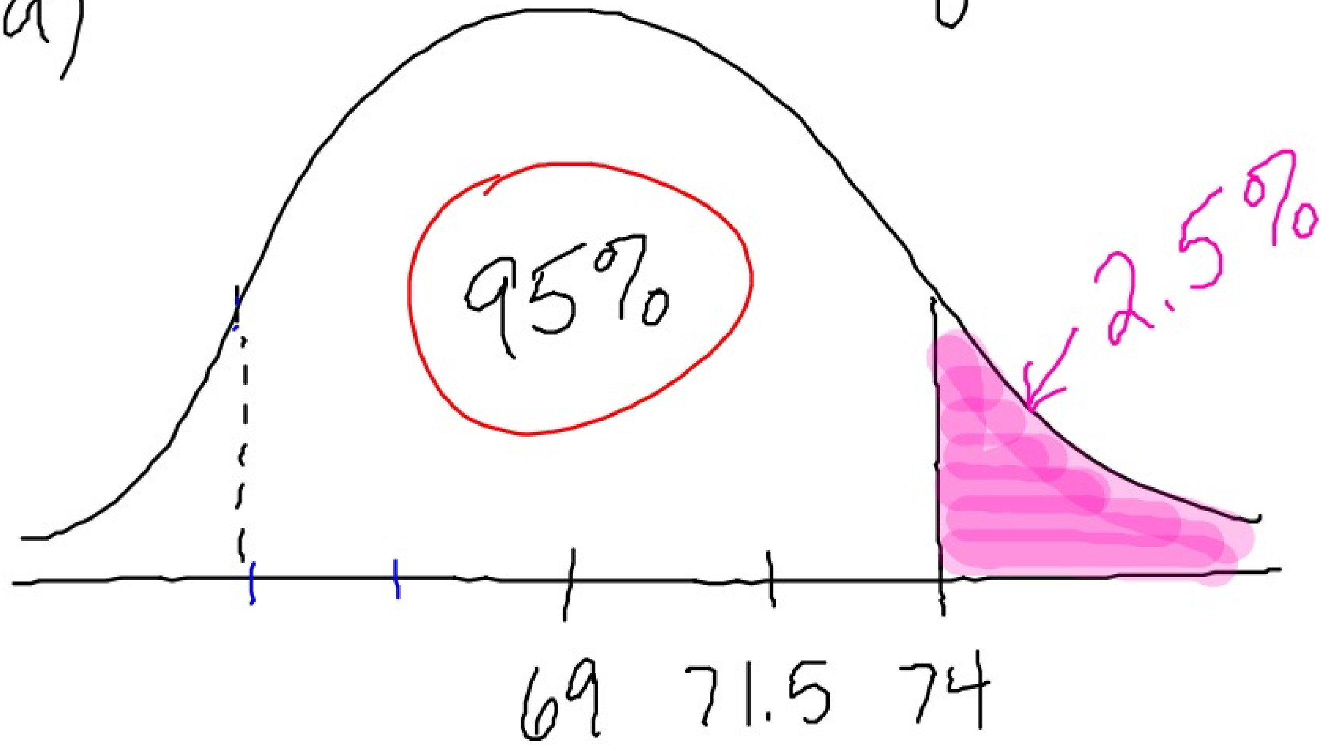
2.41 The Normal density curve with mean 69 and standard deviation 2.5 is shown below.



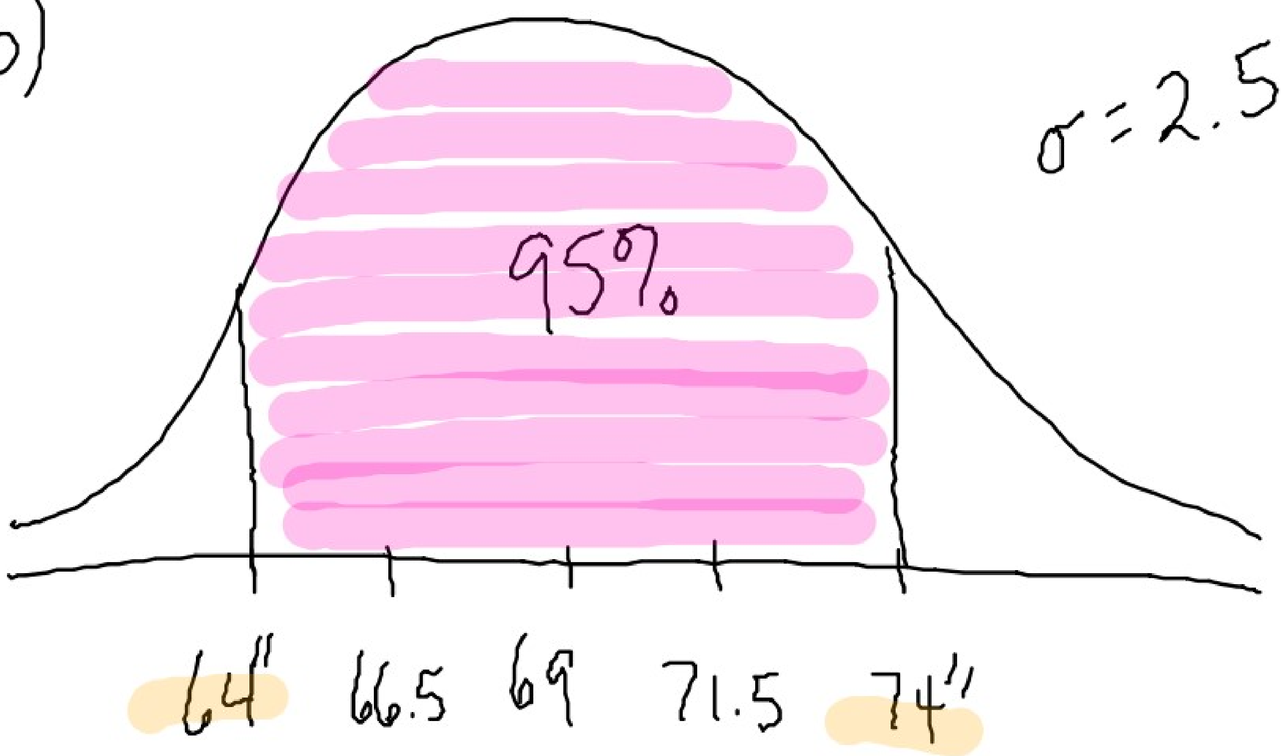
43a)

$\sigma = 2.5$

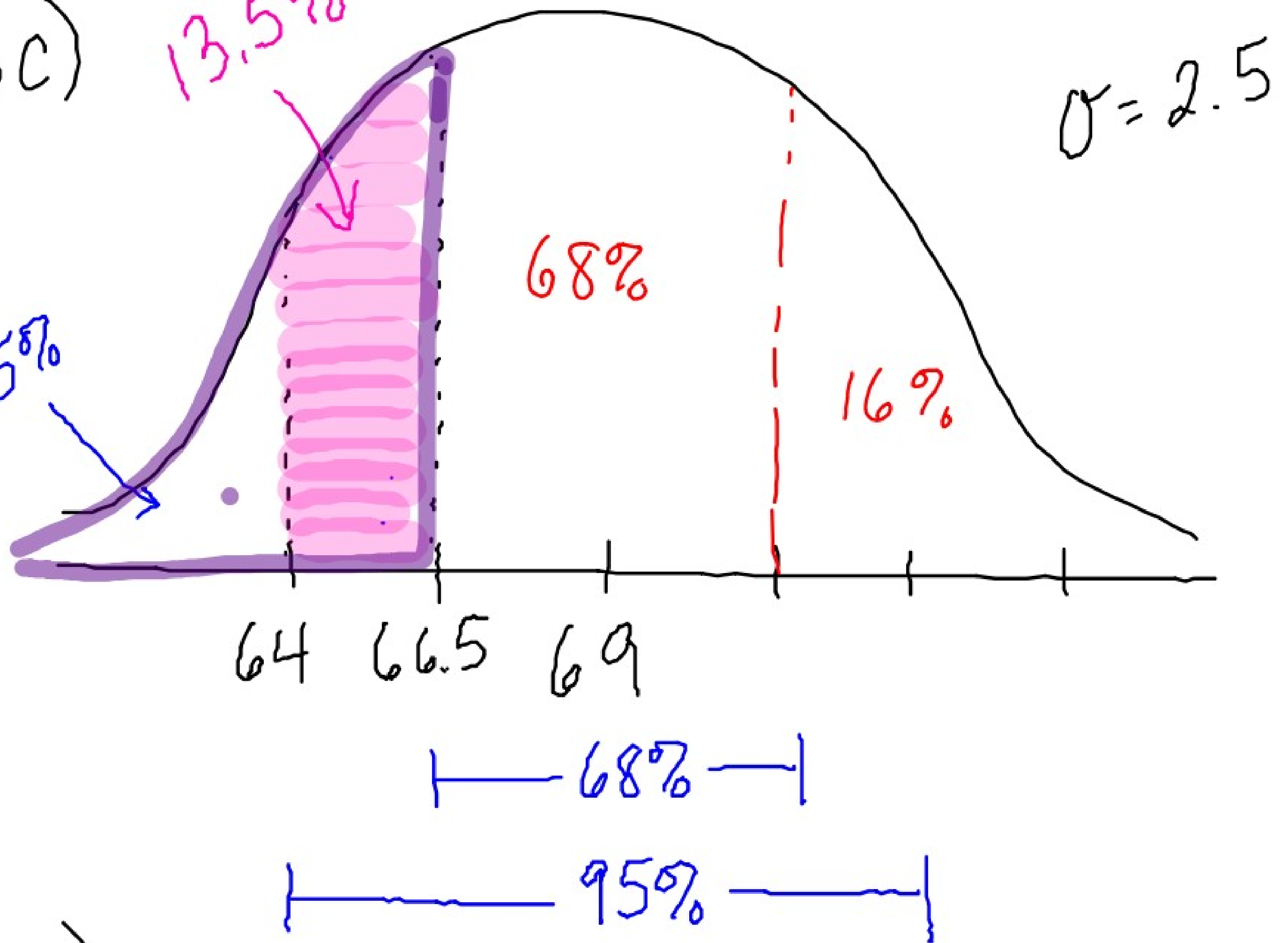


43b)

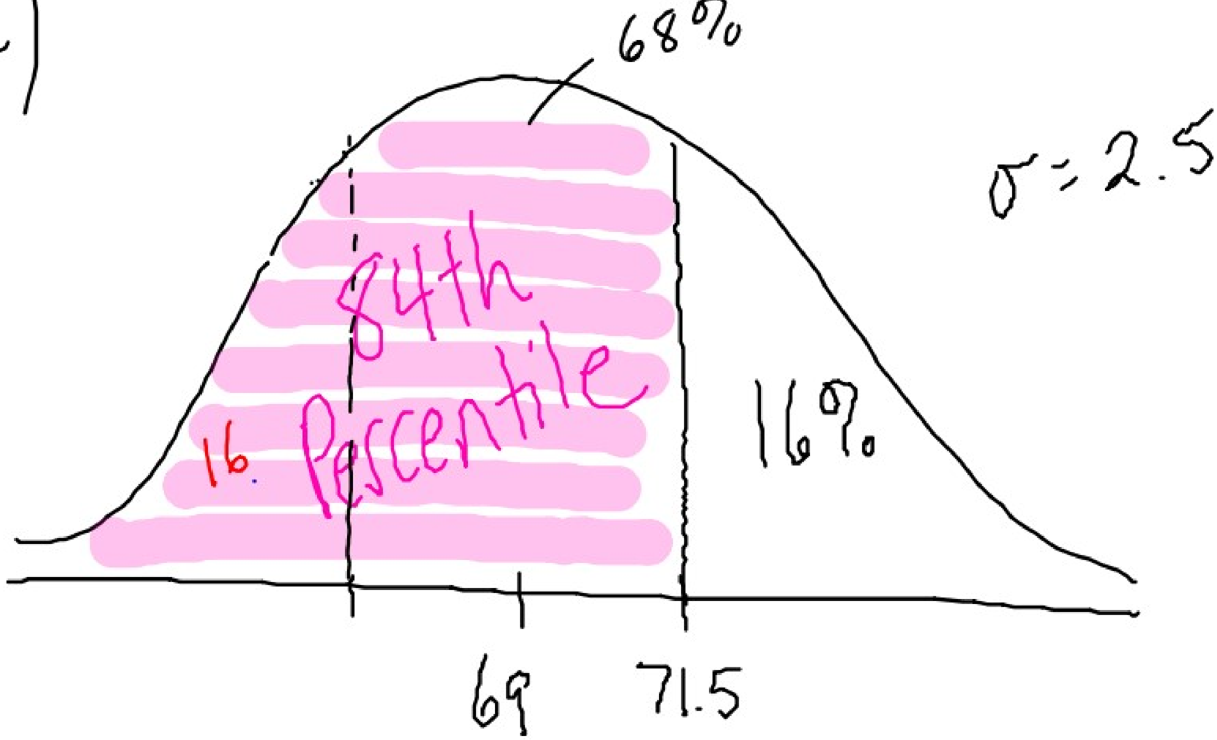
$\sigma = 2.5$



43c)

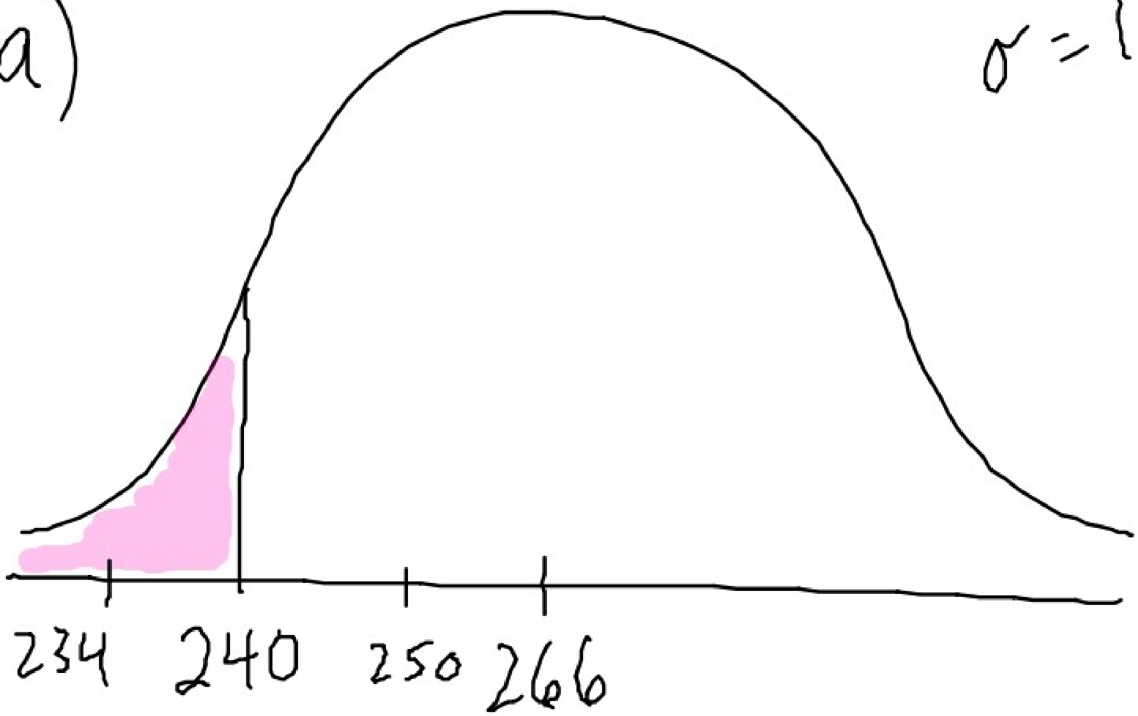

13.5%
σ = 2.5
68%
2.5%
16%
64
66.5
69
68%
95%

43d)


68%
σ = 2.5
84th
Percentile
16.
16%
69
71.5

53a)

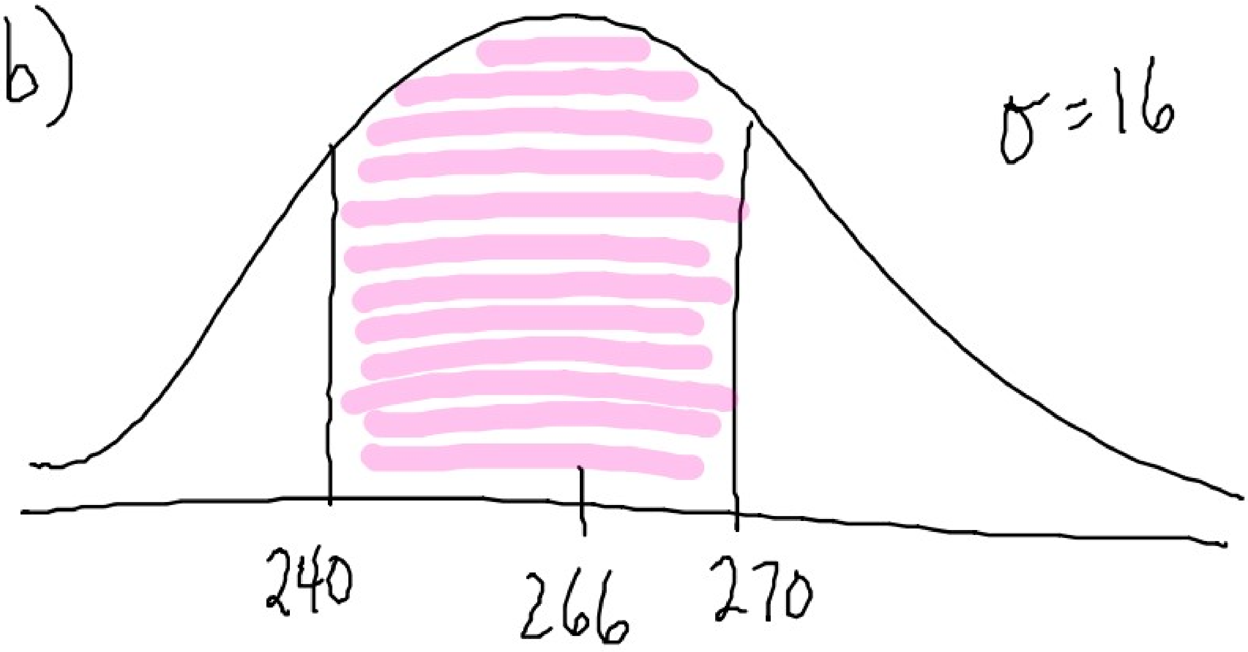
$\sigma = 16$



normalcdf(0, 240, 266, 16) ≈ 5.21%

53b)

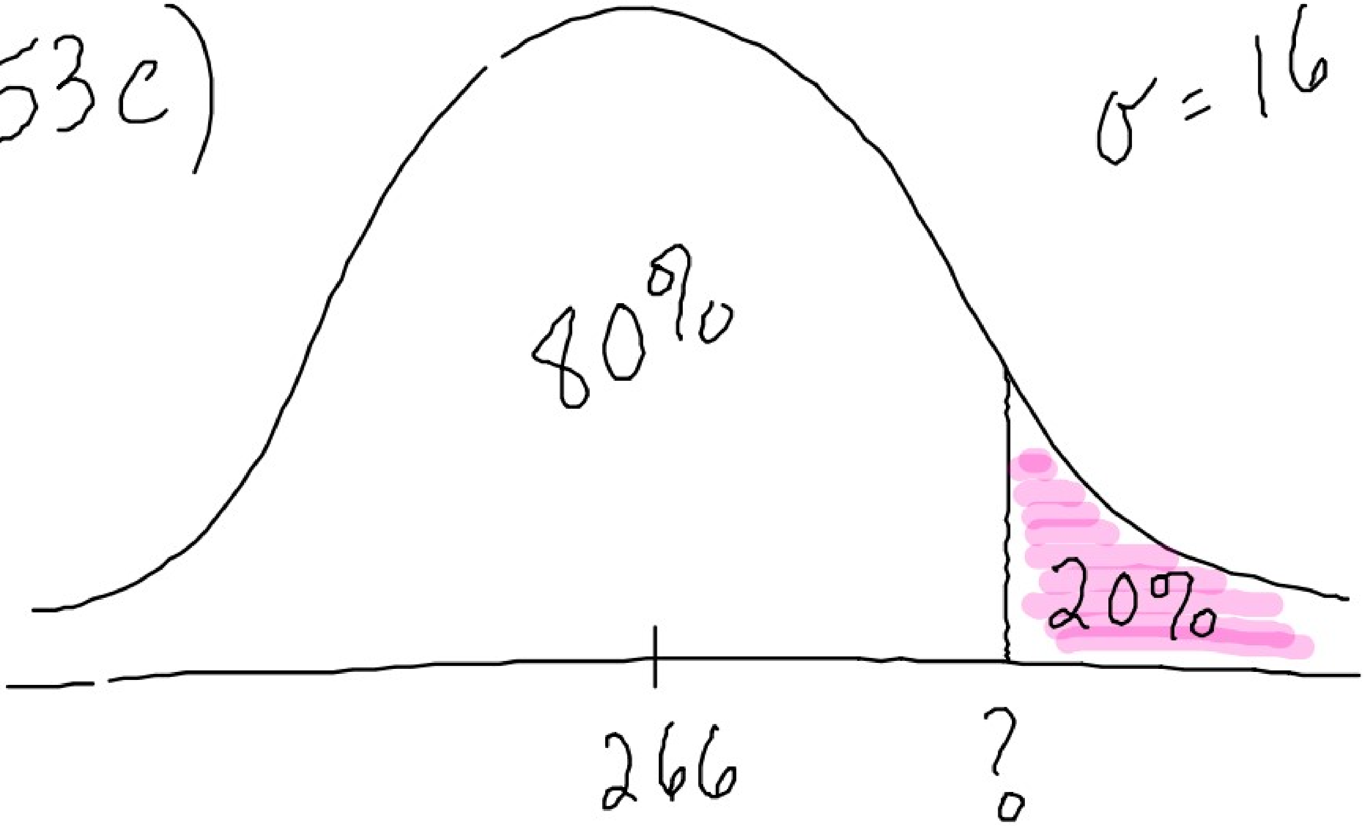
$\sigma = 16$



normalcdf(240, 270, 266, 16) ≈ 54.66%

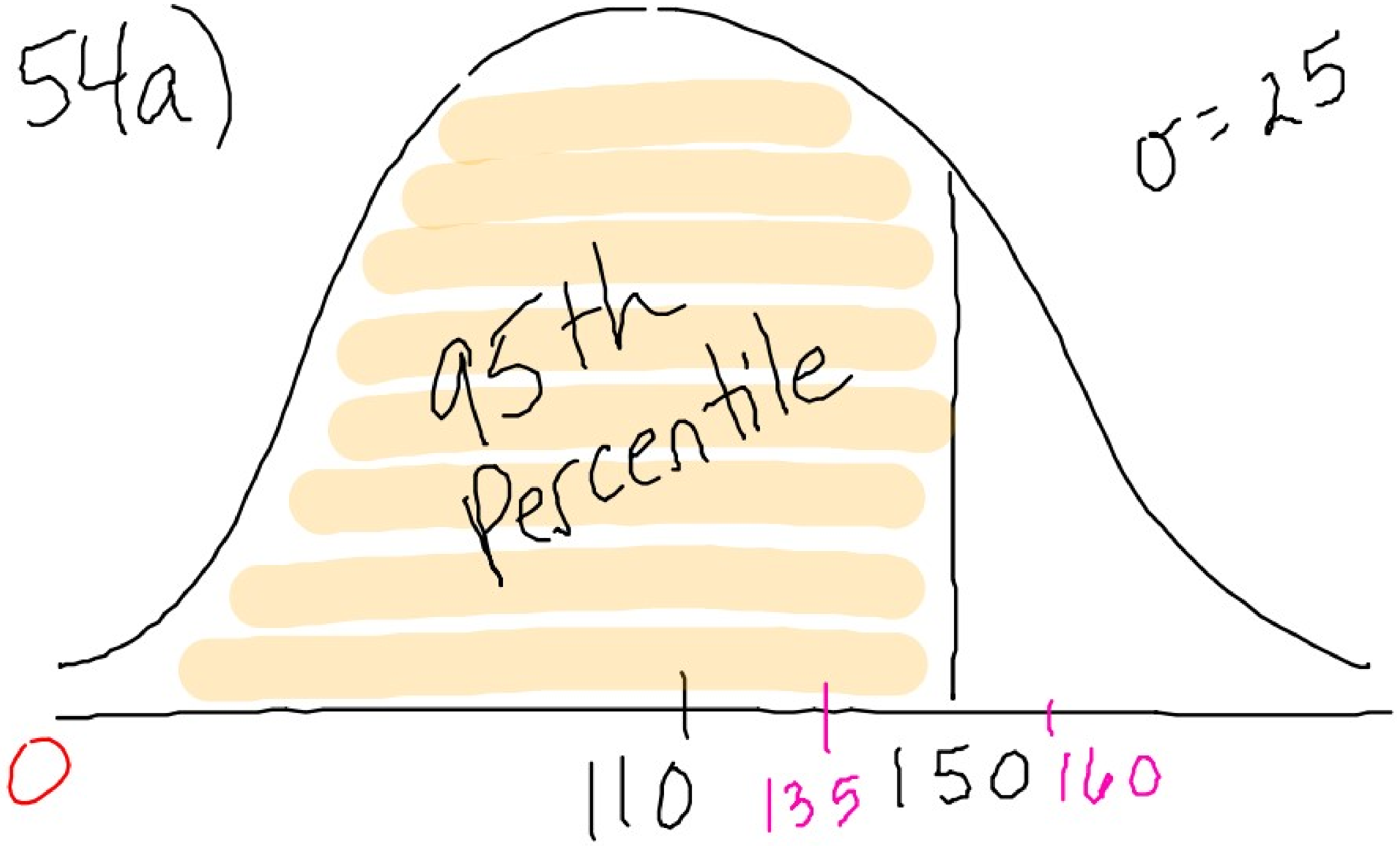
53c)

$\sigma = 16$



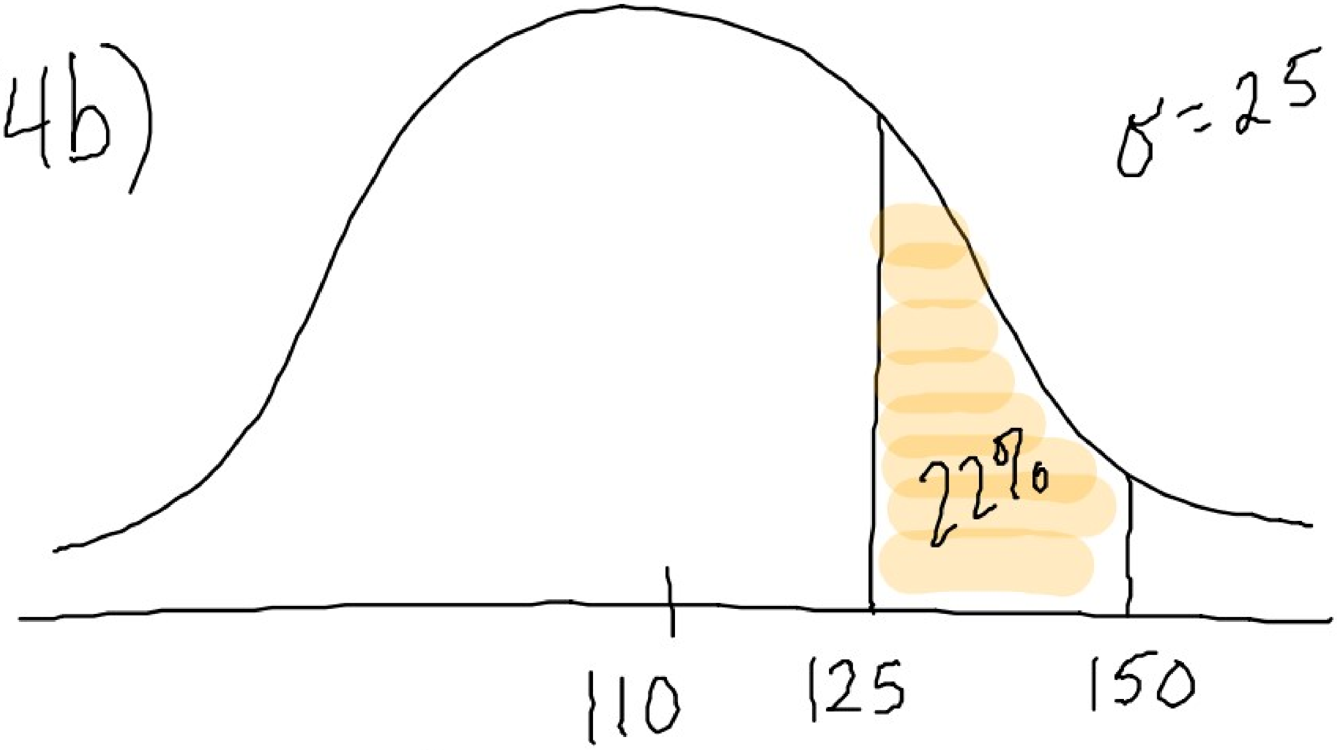
invNorm$(.80, 266, 16) \approx 279$ days

54a)



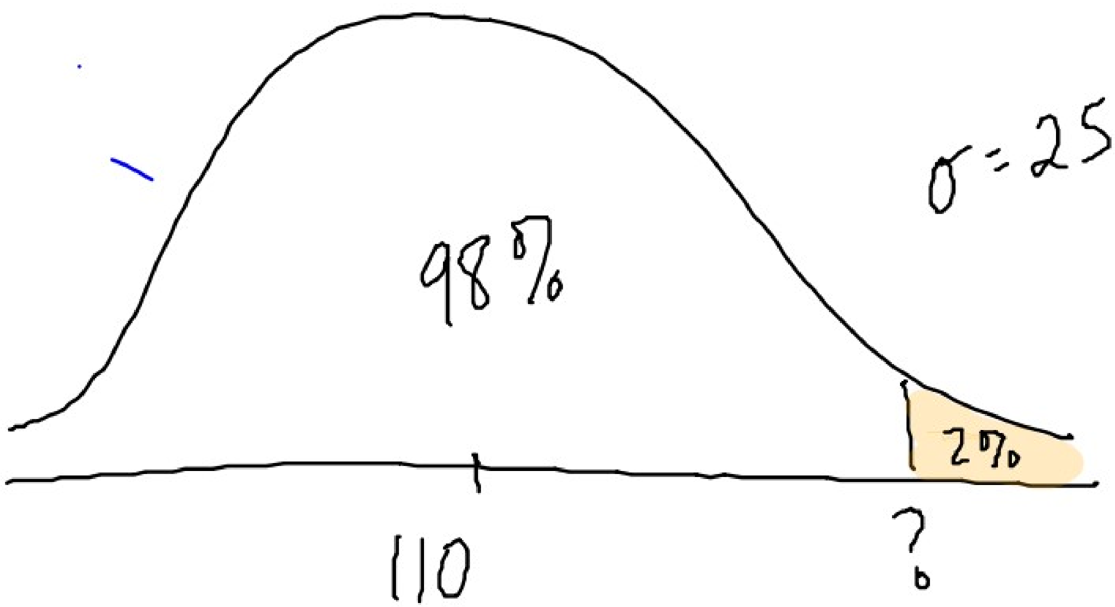
normalcdf $(0, 150, 110, 25) \approx .9452$

54b)



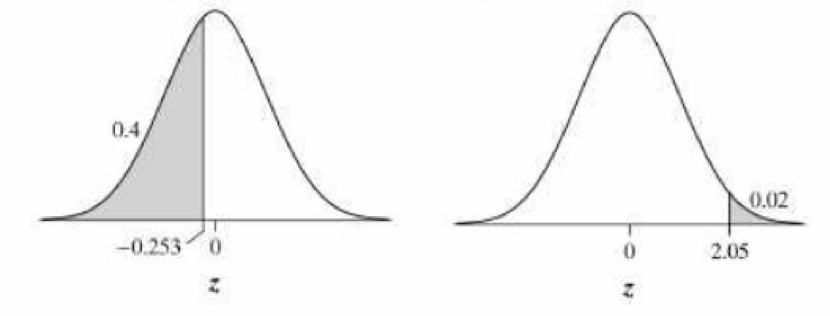
normalcdf(125, 150, 110, 25) ≈ .2195

54c)



invNorm(.98, 110, 25) ≈ 161

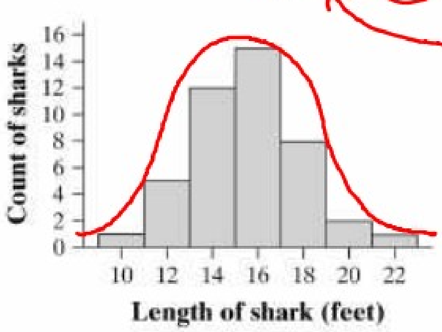
2.62 Use the given information and the graphs below to set up two equations in two unknowns.

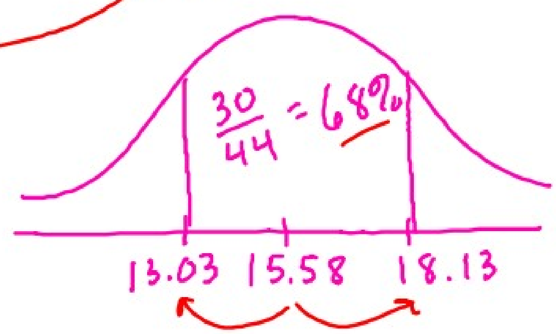


The two equations are $-0.25=\dfrac{1-\mu}{\sigma}$ and $2.05=\dfrac{2-\mu}{\sigma}$. Multiplying both sides of the equations by σ and subtracting yields $-2.3\sigma=-1$ or $\sigma=0.4348$ minutes. Substituting this value back into the first equation we obtain $-0.25=\dfrac{1-\mu}{0.4348}$ or $\mu=1+0.25(0.4348)=1.1087$ minutes.

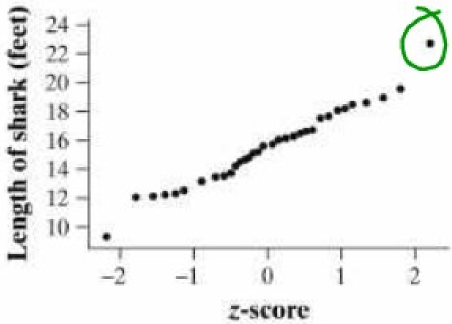
2.63 (a) Descriptive statistics and a histogram are provided below.

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
shlength	44	15.586	2.550	9.400	13.525	15.750	17.400	22.800





The distribution of shark lengths is roughly symmetric with a peak at 16 and varies from 9.4 feet to 22.8 feet. (b) 68.2% of the lengths fall within one standard deviation of the mean, 95.5% of the lengths fall within two standard deviations of the mean, and 100% of the lengths fall within 3 standard deviations of the mean. These are very close to the 68-95-99.7 rule. (c) A Normal probability plot is shown below.

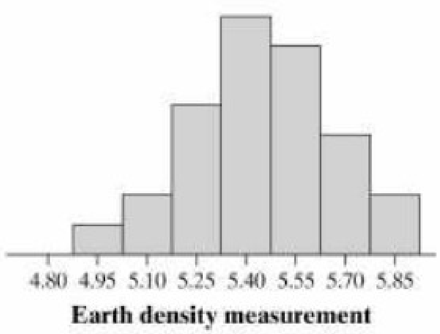


npp

Except for one small shark and one large shark, the plot is fairly linear, indicating that the Normal distribution is appropriate. (d) The graphical display in (a), check of the 68−95−99.7 rule in (b), and Normal probability plot in (c) indicate that shark lengths are approximately Normal.

2.64 (a) Descriptive statistics and a histogram are provided below.

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
density	29	5.4479	0.2209	4.8800	5.2950	5.4600	5.6150	5.8500

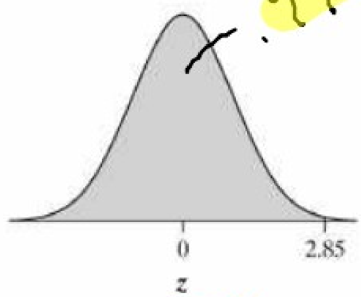


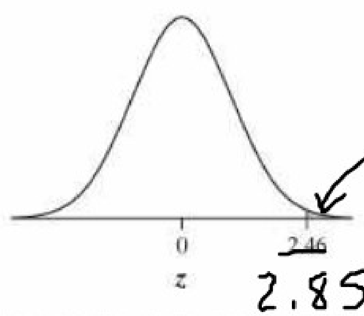
The measurements of the earth's density are roughly symmetric with a mean of 5.45 and varies from 4.88 to 5.85. (b) The densities follow the 68−95−99.7 rule closely—75.86% (22 out of 29) of the densities fall within one standard deviation of the mean, 96.55% (28 out of 29) of the densities fall within two standard deviations of the mean, and 100% of the densities fall within 3 standard deviations of the mean. (c) Normal probability plots from Minitab (left) and a TI calculator (right) are shown below.

47a) b)

.9978

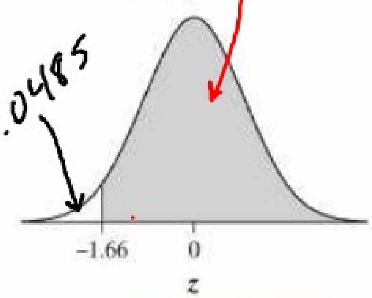
1-.9978 = .0022

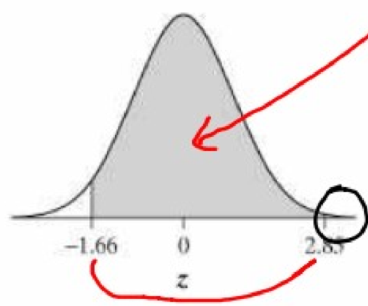




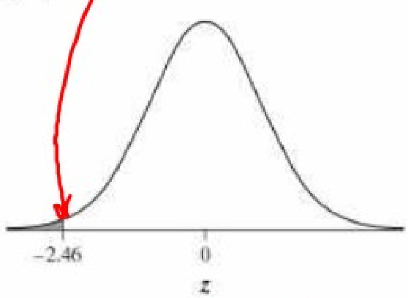
(c) 1 – 0.0485 = 0.9515. The graph is show below (left). (d) 0.9978 – 0.0485 = 0.9493. The graph is show below (right).

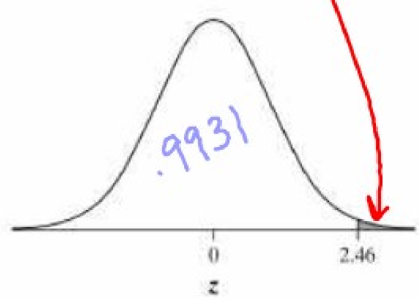
.0485



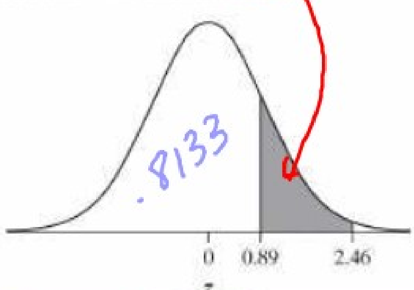


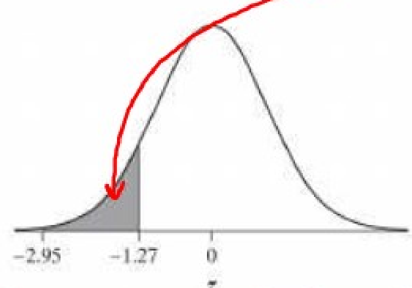
2.48 (a) 0.0069. The graph is show below (left). (b) 1 − 0.9931 = 0.0069. The graph is shown below (right).





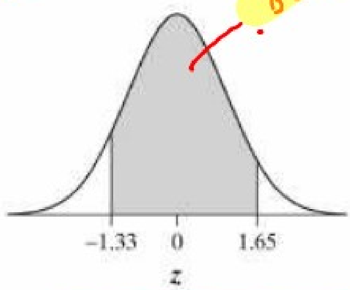
(c) 0.9931 − 0.8133 = 0.1798. The graph is shown below (left). (d) 0.1020 − 0.0016 = 0.1004. The graph is shown below (right).



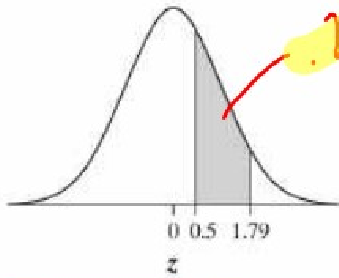


2.49 (a) 0.9505 – 0.0918 = 0.8587. The graph is shown below (left). (b) 0.9633 – 0.6915 = 0.2718. The graph is shown below (right).

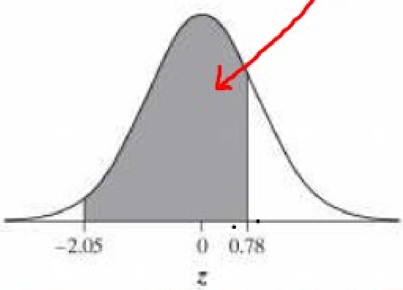
49a) .9505 - .0918 = .8587

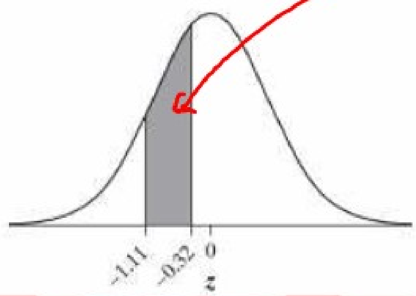


49b) .9633 - .6915 = .2718



2.50 (a) $0.7823 - 0.0202 = 0.7621$. The graph is shown below (left). (b) $0.3745 - 0.1335 = 0.241$. The graph is shown below (right).





2.51 (a) The value that is closest to 0.1000 in Table A is 0.1003. This corresponds to a value of -1.28 for z. (b) The point where 34% of observations are greater is also the $10 - 34 = 66^{th}$ percentile. The value that is closest to 0.6600 in Table A is 0.6591 which corresponds to a z-score of 0.41.

2.52 (a) The value that is closest to 0.6300 in Table A is 0.6293 which corresponds to a z-value of 0.33. (b) If 75% of values are greater than z, then 25% are lower. The value that is closest to 0.2500 in Table A is 0.2514 which corresponds to a z-score of -0.67.

2.53 (a) *State:* Let x = the length of pregnancies. The variable x has a Normal distribution with $\mu = 266$ days and $\sigma = 16$ days. We want the proportion of pregnancies that last less than 240 days. *Plan:* The proportion of pregnancies lasting less than 240 days is shown in the graph below (left).

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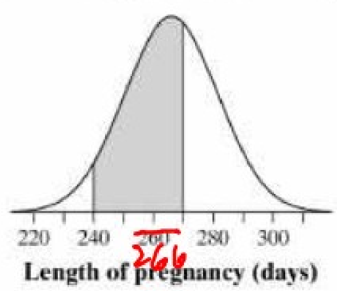
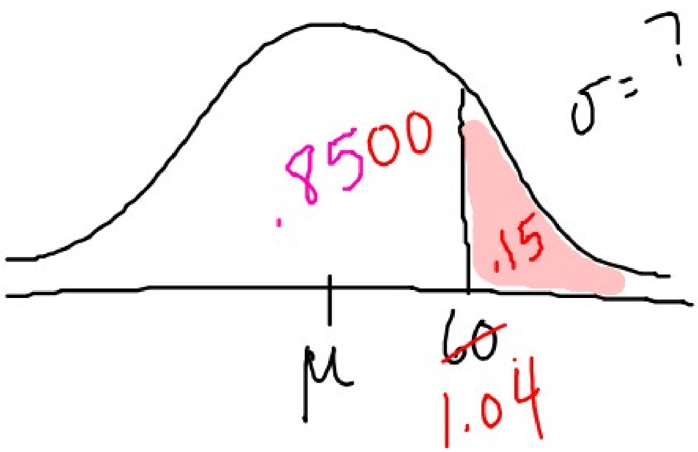
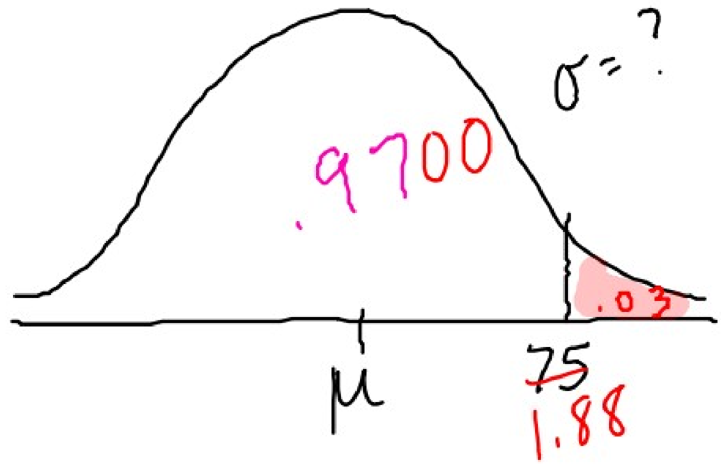


Do: For $x = 240$ we have $z = \dfrac{240 - 266}{16} = -1.63$, so $x < 240$ corresponds to $z < -1.63$. Using table A we see that the proportion of observations less than -1.63 is 0.0516 or about 5.2%. *Conclude:* About 5.2% of pregnancies last less than 240 days which means that 240 is approximately the 5^{th} percentile.

61) Flight Times (Minutes):





$$Z = \frac{X - \mu}{\sigma}$$

$$1.04 = \frac{60 - \mu}{\sigma} \qquad \Big| \qquad 1.88 = \frac{75 - \mu}{\sigma}$$

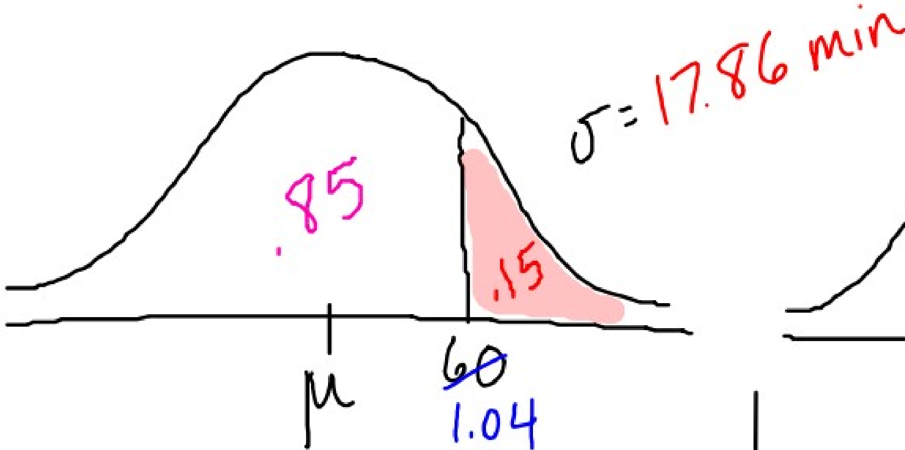
$$1.04\sigma = 60 - \mu \qquad \Big| \qquad 1.88\sigma = 75 - \mu$$

$$\mu = 60 - 1.04\sigma \qquad \Big| \qquad \mu = 75 - 1.88\sigma$$

$$60 - 1.04\sigma = 75 - 1.88\sigma$$

$$\sigma = 17.86 \text{ min}$$

61) Flight Times (Minutes):



$$Z = \frac{X - \mu}{\sigma}$$

$$\frac{1.04}{1} = \frac{60 - \mu}{17.86}$$

$$\mu = 41.42 \text{ min}$$

$$Z = \frac{X - \mu}{\sigma}$$

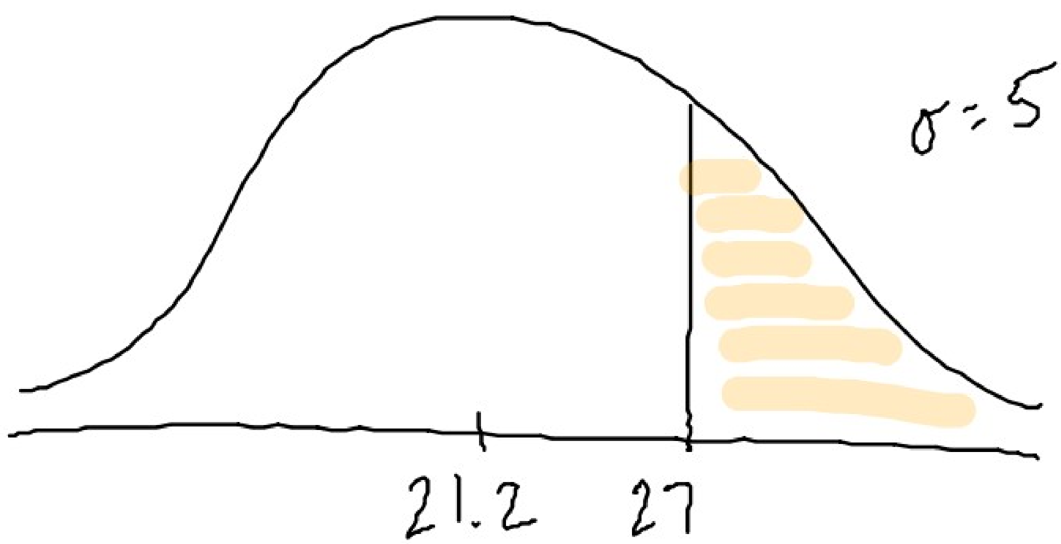
$$\frac{1.88}{1} = \frac{75 - \mu}{17.86}$$

$$\mu = 41.42 \text{ min}$$

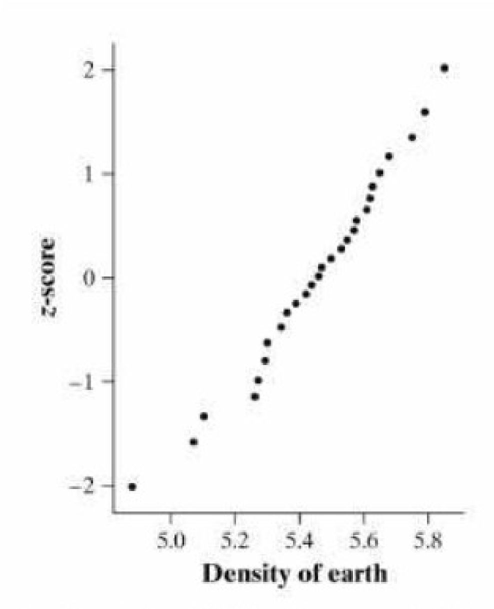
67a) $P(>27) = \frac{149{,}164}{1{,}300{,}599} = 11.47\%$

67b) $P(\geq 27) = \frac{199474}{1{,}300{,}599} \approx 15.34\%$

67c)



normalcdf (27, 100, 21.2, 5) = 12.3%



The Normal probability plot is roughly linear, indicating that the densities are approximately Normal. (d) The graphical display in (a), the 68-95-99.7 rule in (b) and the Normal probability plot in (c) all indicate that these measurements are approximately Normal.

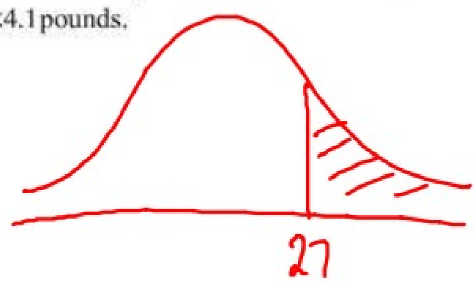
2.65 The plot is nearly linear. Because heart rate is measured in whole numbers, there is a slight "step" appearance to the graph.

2.66 The shape of the Normal probability plot suggests that the data are right-skewed. This can be seen in the steep, nearly vertical section in the lower left—these numbers were less spread out than they should be for Normal data—and the three apparent outliers that deviate from the line in the upper right; these were much larger than they would be for a Normal distribution.

2.67 (a) The percent of scores above 27 is $\frac{149{,}164}{1{,}300{,}599} = 0.1147$ or about 11.47%. (b) The percent of scores greater than or equal to 27 is $\frac{149{,}164 + 50{,}310}{1{,}300{,}599} = 0.1534$ or about 15.34%. (c) For the normal distribution with $\mu = 21.2$ and $\sigma = 5.0$, the percent of observations greater than 27 corresponds to the percent of observations above $z = \frac{27 - 21.2}{5} = 1.16$ on the Standard Normal curve. Using Table A, this proportion is 0.1230. So about 12.30% of scores would be 27 or higher.

2.68 Women's weights are skewed to the right: This makes the mean higher than the median, and it is also revealed in the differences $M - Q_1 = 133.2 - 118.3 = 14.9$ pounds and $Q_3 - M = 157.3 - 133.2 = 24.1$ pounds.

normalcdf



2.69 d

2.70 c

2.71 b