

Directions: *Work on these sheets.*

**Part 1: Multiple Choice.** *Circle the letter corresponding to the best answer.*

1. A dealer in the Sands Casino in Las Vegas selects 40 cards from a standard deck of 52 cards. Let  $Y$  be the number of red cards (hearts or diamonds) in the 40 cards selected. Which of the following best describes this setting:
  - (a)  $Y$  has a binomial distribution with  $n = 40$  observations and probability of success  $p = 0.5$ .
  - (b)  $Y$  has a binomial distribution with  $n=40$  observations and probability of success  $p = 0.5$ , provided the deck is shuffled well.
  - (c)  $Y$  has a binomial distribution with  $n=40$  observations and probability of success  $p = 0.5$ , provided after selecting a card it is replaced in the deck and the deck is shuffled well before the next card is selected.
  - (d)  $Y$  has a normal distribution with mean  $p = 0.5$ .
  
2. In a certain large population, 40% of households have a total annual income of over \$70,000. A simple random sample is taken of 4 of these households. Let  $X$  be the number of households in the sample with an annual income of over \$70,000 and assume that the binomial assumptions are reasonable. What is the mean of  $X$ ?
  - (a) 1.6
  - (b) 28,000
  - (c) 0.96
  - (d) 2, since the mean must be an integer
  - (e) The answer cannot be computed from the information given.
  
3. The probability that a three-year-old battery still works is 0.8. A cassette recorder requires four working batteries to operate. The state of batteries can be regarded as independent, and four three-year-old batteries are selected for the cassette recorder. What is the probability that the cassette recorder operates?
  - (a) 0.9984
  - (b) 0.8000
  - (c) 0.5904
  - (d) 0.4096
  - (e) The answer cannot be computed from the information given.
  
4. Twenty percent of all trucks undergoing a certain inspection will fail the inspection. Assume that trucks are independently undergoing this inspection, one at a time. The expected number of trucks inspected before a truck fails inspection is
  - (a) 2
  - (b) 4
  - (c) 5
  - (d) 20
  - (e) The answer cannot be computed from the information given.

5. Two percent of the circuit boards manufactured by a particular company are defective. If circuit boards are randomly selected for testing, the probability that the number of circuit boards inspected before a defective board is found is greater than 10 is
- (a)  $1.024 \times 10^7$
  - (b)  $5.12 \times 10^7$
  - (c) 0.1829
  - (d) 0.8171
  - (e) The answer cannot be computed from the information given.

**Part 2: Free Response**

*Answer completely, but be concise. Write sequentially and show all steps.*

A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

6. Define the random variable being measured.  $X =$

What kind of distribution does  $X$  have?

7. Calculate the mean and standard deviation of  $X$ .

8. Determine the probability that exactly 80 subjects experience headache relief with this remedy.

9. What is the probability that between 75 and 90 (inclusive) of the patients will obtain relief? Justify your method of solution.

The Ferrells have three children: Jennifer, Jessica, and Jaclyn. If we assume that a couple is equally likely to have a girl or a boy, then how unusual is it for a family like the Ferrells to have three children who are all girls. Let  $X$  = number of girls (in a family of three children).

10. Construct a pdf (probability distribution function) table for the variable  $X$ .


11. Construct a pdf histogram for  $X$ .

12. Construct a cdf (cumulative distribution function) table for  $X$ .

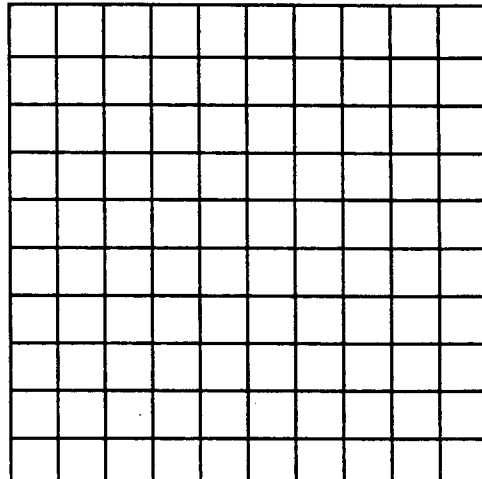

13. Construct a cdf histogram for  $X$ .

14. What is the probability that a family like the Ferrells would have three children who are all girls?

A survey conducted by the Harris polling organization discovered that 63% of all Americans are overweight. Suppose that a number of randomly selected Americans are weighed.

15. Find the probability that 18 or more of the 30 students in a particular adult Sunday School class are overweight.
  
  
  
  
  
  
  
  
  
  
16. How many Americans would you expect to weigh before you encounter the first overweight individual?
  
  
  
  
  
  
  
  
  
  
17. What is the probability that it takes more than 5 attempts before an overweight person is found?
  
  
  
  
  
  
  
  
  
  
18. Construct the cumulative distribution table (stop at  $n = 6$ ) for the number of Americans weighed before an overweight person was found.

19. Sketch a cumulative distribution histogram (stop at  $n = 6$ ) for the table you constructed in the previous problem. Don't forget to label the axes.

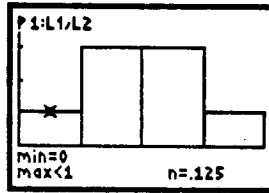


(1) c (2) a (3) d (4) c (5) d (6)  $X$  = number of patients whose headaches are cured.  $X$  is  $B(n, p) = B(100, .8)$ . (7)  $\mu = np = 100(.8) = 80$ .  $\sigma = \sqrt{(80(.2))} = \sqrt{16} = 4$ . (8)  $P(X = 80) = \text{binompdf}(100, .8, 80) = .0993$ . (9)  $P(75 \leq X \leq 90) = \text{binomcdf}(100, .8, 90) - \text{binomcdf}(100, .8, 74) = .9976664 - .087475 = .9102$ .

(10)

$X$	0	1	2	3
$P(X)$	.125	.375	.375	.125

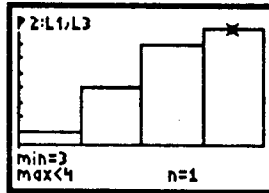
(11) See graph.



(12)

$X$	0	1	2	3
$P(X)$	.125	.5	.875	1

(13) See graph.



(14)  $P(X = 3) = 1/8 = .125 = \text{binompdf}(3, .5, 3)$ . (15)  $X$  is  $B(30, .63)$ .  $P(X \geq 18) = 1 - \text{binomcdf}(30, .63, 17) = .7055$ . (16) The random variable,  $Y$  = number of Americans weighed before an overweight individual is observed, is geometric.  $\mu_Y = 1/p = 1/.63 = 1.5873$ . (17)  $P(Y > 5) = (1 - p)^n = (.37)^5 = .0069$ .

(18)

$X$	1	2	3	4	5	6
$P(X)$	.63	.8631	.9494	.9813	.9931	.9974

(19) See graph.

