Mathematics 100 6 November 2006

The Distribution of the Sample Mean $\overline{X}$ In Simple Random Sampling : An Example of Sampling Distributions

Let us reconsider the population of size $N = 6$ from the second lesson, which is as follows:

2 6 8 10 10 12

Since we know the *entire* population [starting soon, that will not be the case again], we can find the values of the parameters μ and σ, which are $\mu = 48/6 = 8$ and $\sigma = \sqrt{64/6} \doteq 3.27$.

Let us first consider simple random samples (SRS) of size $n = 2$. We again list them below, along with their means ($\bar{x}$), the *estimates* of μ:

Sample	$\overline{X}$
2, 6	4
2, 8	5
2, 10	6
2, 10	6
2, 12	7
6, 8	7
6, 10	8
6, 10	8
6, 12	9
8, 10	9
8, 10	9
8, 12	10
10, 10	10
10, 12	11
10, 12	11

Thus, we have the following *sampling distribution*:

Distribution of $\overline{X}$ for n = 2

$\overline{X}$	Frq
4	1
5	1
6	2
7	2
8	2
9	3
10	2
11	2

(Continued)

The mean of this distribution, $\mu_{\bar{x}}$, is

$$\mu_{\bar{x}} = \frac{(1)(4)+(1)(5)+(2)(6)+\cdots+(2)(11)}{15} = \frac{120}{15} = 8 = \mu$$

illustrating that the mean value [or expected value, E($\bar{x}$)] of the sampling distribution is the population mean, that is,

$$\mu_{\bar{x}} = \mu$$

The variance of the distribution, $\sigma_{\bar{x}}^{\,2}$, called the *sampling variance*, is

$$\text{Var}\,(\bar{x}) = \sigma_{\bar{x}}^{\,2} = \frac{1(4-8)^2+1(5-8)^2+2(6-8)^2+\cdots+2(11-8)^2}{15} = \frac{64}{15}$$

and so $\sigma_{\bar{x}}$, called the *standard error* of the mean, is

$$\sigma_{\bar{x}} = \sqrt{\frac{64}{15}} \doteq 2.07$$

Let us now use this sampling distribution to confirm that

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

by substituting in the values. We have

$$\sqrt{\frac{64}{15}} \stackrel{?}{=} \frac{\sqrt{64/6}}{\sqrt{2}}\sqrt{\frac{6-2}{6-1}}$$

$$= \sqrt{\frac{64}{12}}\sqrt{\frac{4}{5}}$$

$$= \sqrt{\frac{64}{15}}$$

which verifies the result.

(Continued)

Since $N = 6$ for our population, there are six possible sampling distributions: for $n = 1$ (which is just the population distribution), for $n = 2$ (which we have discussed), and for $n = 3$, 4, 5, and 6. All six are given below.

Sampling Distributions of $\overline{X}$

For $n = 1$:

$\bar{x}$	frq
2	1
6	1
8	1
10	2
12	1

For $n = 2$:

$\bar{x}$	frq
4	1
5	1
6	2
7	2
8	2
9	3
10	2
11	2

For $n = 3$:

$\bar{x}$	frq
5.3	1
6	2
6.6	3
7.3	2
8	4
8.6	2
9.3	3
10	2
10.6	1

For $n = 4$:

$\bar{x}$	frq
6.5	2
7	2
7.5	3
8	2
8.5	2
9	2
9.5	1
10	1

For $n = 5$:

$\bar{x}$	frq
7.2	1
7.6	2
8.0	1
8.4	1
9.2	1

For $n = 6$:

$\bar{x}$	frq
8	1

Notice the increasing precision as n increases. For example, as we go from the sampling distribution for $n = 2$ to $n = 3$, the range of possible values for the statistic decreases from 11 – 4 = 7 to 10.6 – 5.3 = 5.3. Another way of saying this is that the sampling variance drops from 64/15 to 64/30, or the standard error of the mean drops from their square roots, from about 2.07 to 1.46. A complete summary is given below.

n	1	2	3	4	5	6
$\mu_{\bar{x}}$	8	8	8	8	8	8
$\sigma_{\bar{x}}$	3.27	2.07	1.46	1.03	0.65	0
range	10	7	5.3	3.5	2	0